

MATH 2270-2, SPRING 2006, PRACTICE FINAL

1. Go over Exam 1, Exam 2, and Exam 3 problems for the old materials (Chapter 1 - Chapter 7).

2. For the recent materials(Chapter 7.4 and 8), study the following problems and related HW problems.

(1) Which of the following matrices are diagonalizable or orthogonally diagonalizable?

For such matrices, write them as SDS^{-1} or SDS^T .

a. $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, b. $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, c. $C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

(2) Consider the linear transformation $L(\vec{x}) = A\vec{x}$, where $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

a. Find the singular values of A .

b. Find orthonormal vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ in \mathbb{R}^3 such that $L(\vec{v}_1), L(\vec{v}_2), L(\vec{v}_3)$ are orthogonal.

c. Find the SVD(Singular Value Decomposition) of A .

(3) Define a map $L : P_2 \rightarrow P_2$ by $L(f(x)) = f(x) - f'(x)$.

a. Show that L is linear.

b. Find the determinant of L .

c. Is L an isomorphism? Explain why.

d. Is L diagonalizable or orthogonally diagonalizable? If then, find a basis of P_2 such that L is represented as a diagonal matrix.

e. Is there a basis of P^2 such that the image vectors of basis vectors under L are orthogonal to each other? Explain why or why not.

(4) For $A = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -3 & 2 \end{bmatrix}$,

a. find A^5 ,

b. find $\lim_{t \rightarrow \infty} A^t$,

⟨ True or False questions on Chapter 7 and 8 ⟩

For TF questions on the old materials, refer Exam 1, Exam 2, and Exam 3.

Determine whether the following statement is True or False.

- (1) If an $n \times n$ matrix A has eigenvalue 0, then A has rank n .
- (2) If A is a matrix such that $A\vec{v} = 3\vec{v}$ and $A\vec{w} = 2\vec{w}$, then $\vec{v} \cdot \vec{w} = 0$.
- (3) If A is a matrix with two independent eigenvectors \vec{v} and \vec{w} , then $\vec{v} \cdot \vec{w} = 0$.
- (4) Only symmetric matrices have SVD (Singular Value Decomposition).
- (5) If A is diagonalizable, A^T is also diagonalizable.
- (6) If an invertible matrix A is diagonalizable, A^{-1} is also diagonalizable.
- (7) If A is diagonalizable, then it is orthogonally diagonalizable.
- (8) Any $n \times m$ matrix A has an orthonormal basis $\vec{v}_1, \dots, \vec{v}_m$ of \mathbb{R}^m for A such that the images $A(\vec{v}_1), \dots, A(\vec{v}_m)$ in \mathbb{R}^n are orthogonal.
- (9) If the singular values of a 2×2 matrix A are 3 and 4, then there must exist a unit vector \vec{u} in \mathbb{R}^2 such that $\|A\vec{u}\| = 4$.
- (10) If A is a symmetric matrix such that \vec{v} and \vec{w} are eigenvectors associated to distinct eigenvalues, then $\vec{v} \cdot \vec{w} = 0$.
- (11) Every symmetric matrix has non-negative eigenvalues.