MATH 2270-2, SPRING 2006, PRACTICE EXAM 2

⟨ Strongly suggested HW problems ⟩

Sec. 4.1 – 2, 21, 22, 20, 24

Sec. 4.2 – 6, 42, 60, 52 (Go over the rank-nullity theorem (Def. 4.2.1), and the properties of Isomorphisms (Fact 4.2.4).)

Sec. 4.3 – 6, 20

Sec. 5.1 – 10, 29, 31 (Go over the Pythagorean theorem (Fact 5.1.9) and the Cauchy-Schwartz inequality (Fact 5.1.11).)

Sec. 5.2 – 19, 32, 33

Sec. 5.3 – 4, 40, 52, 56

Sec. 5.4 – 1, 30, 31

⟨ True or False questions on Chapter 4 and 5 ⟩

Determine whether the following statement is True or False.

1. The space P_3 is isomorphic to the space of all 4×4 diagonal matrices.
2. If T is a linear transformation from P_6 to $\mathbb{R}^{2\times 2}$, then the kernel of T must be 3-dimensional.
3. If the kernel of a linear transformation $T : P_4 \to P_4$ is $\{0\}$, then T must be an isomorphism.
4. If the image of a linear transform $T : V \to V$ is all of V, then T must be an isomorphism.
5. For a linear transform $T : V \to V$, the intersection of $\mathbf{im}(T)$ and $\mathbf{ker}(T)$ must be $\{0_V\}$.
6. The kernel of every orthogonal matrix is $\{\vec{0}\}$.
7. The nullity of every nonzero symmetric matrix is 0.
8. If $A^T A = AA^T$ for an $n \times n$ matrix A, then A must be symmetric.
9. For an $n \times m$ matrix A such that $A^T A = I_m$, $A\vec{e}_1$ and $A\vec{e}_2$ are perpendicular to each other.
10. For an $n \times m$ matrix A such that $A^T A = I_m$, $A\vec{e}_1$ and $A\vec{e}_2$ are linearly independent.
11. A square matrix A which is both orthogonal and symmetric satisfies $A^2 = I_n$.
12. A square matrix A which is both orthogonal and skew-symmetric satisfies $A^2 = -I_n$.
13. A square matrix A which is symmetric and skew-symmetric satisfies $A = O$.

Date: March 3, 2006.