

2270-2, LINEAR ALGEBRA 02/03/05
TRUE OR FALSE PROBLEMS IN CHAPTER 3

Determine whether the following statement is True or False.

1. The image of a 3×4 matrix is a subspace of \mathbb{R}^4 .
2. If $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ are linearly independent vectors in \mathbb{R}^n , then they must form a basis of \mathbb{R}^n .
3. There exists a 5×4 matrix whose image consists of all of \mathbb{R}^5 .
4. The kernel of any invertible matrix consists of the zero vector only.
5. If $2\vec{v}_1 + 3\vec{v}_2 + 4\vec{v}_3 = 5\vec{v}_1 + 6\vec{v}_2 + 7\vec{v}_3$, then $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent.
6. If $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ and $\vec{w}_1, \vec{w}_2, \dots, \vec{w}_m$ are any two bases of a subspace V of \mathbb{R}^{10} , then $n = m$.
7. If A is a 5×6 matrix of rank 4, then the nullity(= $\dim(\ker(A))$) of A is 1.
8. Vectors, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ form a basis of \mathbb{R}^3 .
9. Vectors, $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$, $\begin{bmatrix} 9 \\ 8 \\ 7 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ -1 \\ -2 \end{bmatrix}$ are linearly independent.
10. If a subspace V of \mathbb{R}^3 contains the standard vectors $\vec{e}_1, \vec{e}_2, \vec{e}_3$, then V must be \mathbb{R}^3 .
11. If vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ span \mathbb{R}^4 , then $n = 4$.
12. If vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ are linearly independent vectors of \mathbb{R}^4 , then $n = 4$.
13. If vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ form a basis of \mathbb{R}^4 , then $n = 4$.
14. If A is an invertible $n \times m$ matrix, then the kernels of A and A^{-1} must be equal.
15. If V is any three-dimensional subspace of \mathbb{R}^5 , then V has infinitely many different bases.