

Name: Solution

Student ID #: \_\_\_\_\_

Each problem of #1-#3 is worth 5 points.

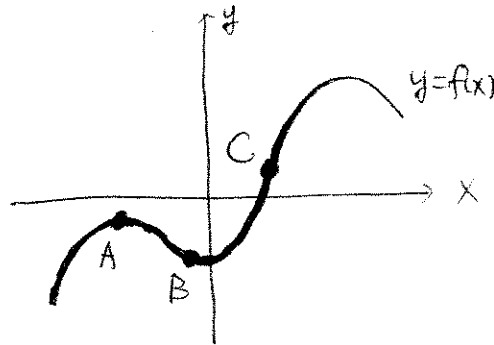
1. Find the derivative function  $f'(x)$  by using the limit definition for  $f(x) = 2x + 1$ .  
(NO credit will be given for using any other formula or estimation over shorter intervals.)

$$f'(x) = \lim_{b \rightarrow x} \frac{f(b) - f(x)}{b - x} = \lim_{b \rightarrow x} \frac{(2b+1) - (2x+1)}{b - x} = \lim_{b \rightarrow x} \frac{2b - 2x}{b - x}$$

$$= \lim_{b \rightarrow x} \frac{2(b-x)}{(b-x)} = \lim_{b \rightarrow x} 2 = \boxed{2}$$

2. Determine the sign (+, - or 0) of  $f$ ,  $f'$  and  $f''$  at each point shown in the graph of  $y = f(x)$ .  
(9 correct: 5 pts, 7-8 correct : 4 pts, 5-6 correct: 3 pts, 3-4 correct: 2 pts, 1-2 correct: 1 pt)

	$f$	$f'$	$f''$
A	-	0	-
B	-	-	+
C	+	+	0



3. Estimate  $f(19)$  when the function  $y = f(x)$  satisfies  $f(21) = 3$  and  $f'(21) = -2$ .

$$f(19) \approx f'(21) \cdot (19 - 21) + f(21)$$

$$= -2 \cdot (-2) + 3$$

$$= 4 + 3$$

$$= \boxed{7}$$

4. (Bonus problem, 1 pt) If  $y = f(x)$  is a continuous function with properties  $f'(x) > 0$  and  $f''(x) = 0$  for all  $x$ , then what is a possible graph of  $y = f(x)$ ? Determine the shape of the graph as, for example, a parabola, a line, a snake-shaped one, etc....and sketch it with considering the concavity and the increasing/decreasing property.

$$\left. \begin{array}{l} f'(x) > 0 \Rightarrow f \text{ is increasing} \\ f''(x) = 0 \Rightarrow \text{no concavity.} \\ \text{So not curved at all} \end{array} \right\} \Rightarrow \text{increasing line}$$

