

Name : _____

Student ID # : _____

Show all of your work to get the full credit. Each problem of #1-#3 is worth 5 points.

1. Find the half-life of the decaying amount $p(t)$ at time t with following $p(t) = 123e^{-t}$.

Find t when $\frac{1}{2} \cdot 123 = 123 \cdot e^{-t}$. Such t is the half-life.

$$\Rightarrow \frac{1}{2} = e^{-t}$$

$$\Rightarrow \ln \frac{1}{2} = -t \cdot \ln e^1$$

$$\ln 2^{-1} = -t$$

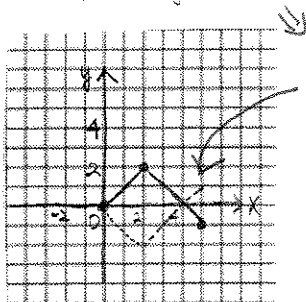
$$\Rightarrow \text{half-life } t = \ln 2$$

$$-\ln 2 = -t$$

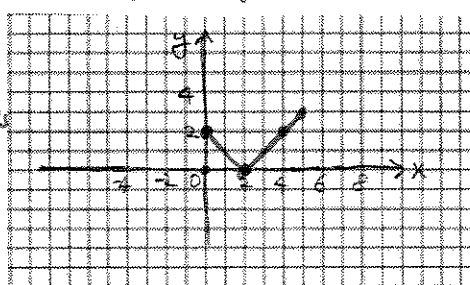
$$\boxed{\ln 2 = t}$$

2. Sketch the graph of $y = -f(x) + 2$ on the right-hand side grid, when the graph of $y = f(x)$ is given on the left.

Graph of $y = f(x)$



Graph of $y = -f(x) + 2$



$y = -f(x)$: reflection about x-axis

$y = -f(x) + 2$
: shifting upward up to 2 units

3. Find $(f \circ g)(x)$ and simplify your answer where $f(x) = (x - 1)^3$ and $g(x) = x + 1$.

$$(f \circ g)(x) = f(g(x)) = f(x+1) = (x+1-1)^3 = (x+1-1)^3 = \boxed{x^3}$$

4. (Bonus problem, 1 pt) Simplify the following as much as possible so that your answer is simply a number without \ln or e .

$$\left(\ln \frac{1}{e^6}\right) \left(e^{\ln \frac{1}{2}}\right) = (\ln e^{-6}) \cdot (e^{\ln \frac{1}{2}}) = (-6 \cdot \ln e) \cdot (e^{\ln \frac{1}{2}}) = -6 \cdot 1 \cdot \left(\frac{1}{2}\right) = \boxed{-3}$$