Name : Solution

Student ID # : ____________

Honor 2201

Fall 2005

Instructor: Bo-Hae Im

EXAM 3

Friday, November 18, 2005

Problem	points	score
1	8	
2	8	
3	8	
4	8	
5	8	
Total	40	

1. Find the definite integral $\int_1^e \frac{1}{x} \cdot \ln(x)dx$.

Let $u = \ln x.$ $\Rightarrow$ $\begin{cases} x=1 \Rightarrow u = \ln 1 = 0 \\ x=e \Rightarrow u = \ln e = 1. \end{cases}$

$du = \frac{1}{x}dx.$

$$\int_1^e \frac{1}{x} \ln x \, dx = \int_0^1 u \, du = \left[\frac{1}{2}u^2\right]_0^1 = \frac{1}{2}\cdot 1^2 - \frac{1}{2}\cdot 0^2$$

$$= \frac{1}{2}$$

(or without changing the limits,

$$\int \frac{1}{x} \ln x \, dx = \int u \, du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$$

So $\int_1^e \frac{1}{x} \ln x \, dx = \left[\frac{1}{2}(\ln x)^2 + C\right]_1^e$

$$= \left(\frac{1}{2}(\ln e)^2 + C\right) - \left(\frac{1}{2}(\ln 1)^2 + C\right)$$

$$= \frac{1}{2}\cdot 1^2 + C - \left(\frac{1}{2}\cdot 0^2 + C\right)$$

$$= \frac{1}{2}$$

)

2. Find the indefinite integral $\int x \cos x dx$.

By integration by parts,

$$\left.\begin{array}{l} \text{let } u = x \\ v' = \cos x. \end{array}\right\} \Rightarrow \begin{array}{l} u' = 1 \\ v = \sin x. \end{array}$$

$$\int x \cos x \, dx = uv - \int u' v \, dx$$

$$= x \cdot \sin x - \int 1 \cdot \sin x \, dx$$

$$= x \cdot \sin x - (-\cos x) + C$$

$$= \boxed{x \cdot \sin x + \cos x + C}$$

3. Find the indefinite integral $\int \sin x \cdot (\cos x + 1)^5 dx$.

Use substitution : $u = \cos x + 1.$

$du = -\sin x \, dx$

$\Rightarrow -du = \sin x \, dx.$

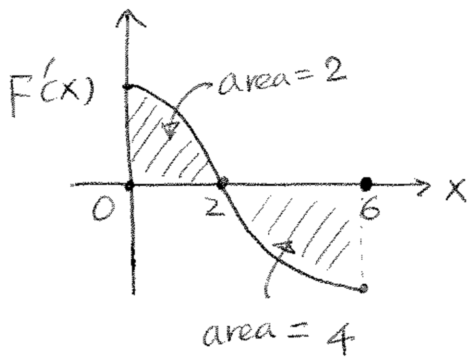
$$\int \sin x \, (\cos x + 1)^5 \, dx = \int u^5 \cdot (-1) \, du$$

$$= -\int u^5 \, du$$

$$= -\frac{1}{6} u^6 + C$$

$$= \boxed{-\frac{1}{6} (\cos x + 1)^6 + C}$$

4. The following is the graph of $F'(x)$ over the interval $[0, 6]$ of the function $F(x)$. If $F(0) = 2$, find the maximum value of $F(x)$ over $[0, 6]$.



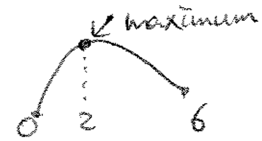
$F'(x) > 0$ for $0 < x < 2$

& $F'(x) < 0$ for $2 < x < 6$.

$\Leftrightarrow$ F is increasing until $x = 2$

& F is decreasing after $x = 2$.

So the shape of F is



By Fundamental theorem of Calculus,

the maximum of F occurs at $x = 2$. &

$$F(2) = F(0) + \int_0^2 F'(x)\,dx$$

$$= 2 + (\quad)$$

$$= 2 + (2 \quad)$$

$$= 2 + (2)$$

$$= \boxed{4}$$

5. Solve the differential equation $P'(x) = -P(x) + 2e^x$, when $P(0) = 2$. (You might need the property $e^a e^b = e^{a+b}$ to simplify your expression while you compute.)

① $e^{\int -(-1)\,dx} = e^{\int 1\,dx} = e^x.$

② $e^x \cdot p' = -e^x p + 2 \cdot e^x \cdot e^x$

$e^x p' + e^x p = 2e^{2x}$

$(p \cdot e^x)' = 2e^{2x}$

③ $p \cdot e^x = \int 2e^{2x}\,dx + C$

$= 2 \cdot \frac{1}{2} e^{2x} + C$

$p \cdot e^x = e^{2x} + C.$

④ $p = \dfrac{e^{2x} + C}{e^x}$

$p(x) = e^x + C \cdot e^{-x}.$

⑤ Since $p(0) = 2$, $p(0) = e^0 + C \cdot e^{-0} = 2$

$\Rightarrow 1 + C \cdot 1 = 2$

$C \cdot 1 = 1$

$\Rightarrow C = 1.$

So the answer is $\boxed{p(x) = e^x + e^{-x}}$