Name : Solution

Student ID # :

Honor 2201

Fall 2005

Instructor: Bo-Hae Im

EXAM 2

Friday, October 21, 2005

Problem	points	score
1 (1)-(3)	10	
1 (4)-(5)	7	
1(6)	5	
2	8	
3	6	
4	4	
Total	40	

1. Let $f(x) = -4x^3 + 12x - 8$ for $-3 \leq x \leq 2$ be a given function.

(1)(3 pts) Find the critical points of f given above if any.

$$f'(x) = -12x^2 + 12 = 0$$

$$-12(x^2 - 1) = 0$$

$$-12(x-1)(x+1) = 0$$

$$\Rightarrow \boxed{x = 1 \quad \text{or} \quad x = -1}$$ are critical points of f.

(2)(4 pts) Find a local maximum and a local minimum of f given above and justify your answer.

$f''(x) = -24x$ & Use the 2nd derivative test!

$f''(1) = -24 < 0.$ ← local max

$f''(-1) = 24 > 0,$ ← local min.

$f(1) = -4 \cdot 1^3 + 12 \cdot 1 - 8 = 0$

& $f(-1) = -4(-1)^3 + 12(-1) - 8 = 4 - 12 - 8 = -16$

$\boxed{\text{So } f \text{ has the local max. } 0 \text{ at } x=1 \text{ \& the local min. } -16 \text{ at } x=-1}$

(3)(3 pts) Find inflection points of f given above if any and justify your answer.

$f''(x) = -24x = 0. \Rightarrow x = 0.$

x		0	
$f''(x)$	+		−

$\Rightarrow f$ changes the concavity at 0.

$\boxed{\text{So } x=0 \text{ is an inflection point of } f}$

(4)(3 pts) Find the global maximum and the global minimum of f given above and justify your answer.

$f(1) = -4\cdot 1^3 + (2\cdot 1) - 8 = -4+12-8 = 0$

$f(-1) = -4\cdot(-1)^3 + 12\cdot(-1) - 8 = 4 - 12 - 8 = -16$

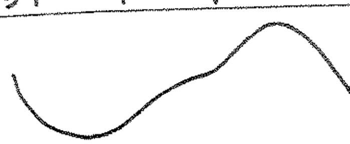
$f(-3) = -4(-3)^3 + 12(-3) - 8 = 4\cdot 27 - 36 - 8 = 108 - 44 = 64$

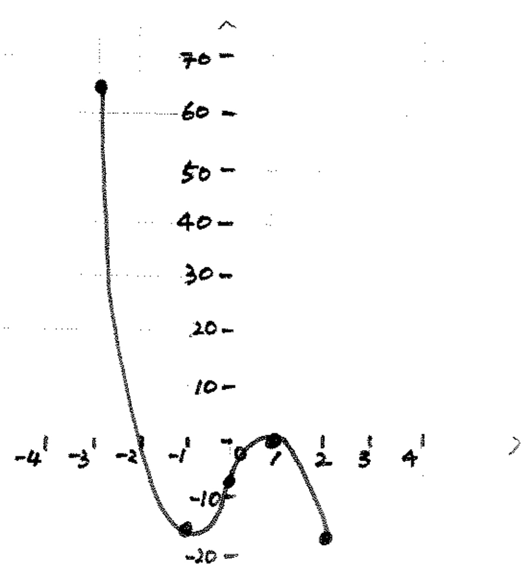
$f(2) = -4\cdot 2^3 + 12\cdot(2) - 8 = -32 + 24 - 8 = -16$

f has the global max 64 at $x=-3$
& the global min -16 at $x=-1$ & $x=2$.

(5)(4 pts) Sketch the graph of f given above by using the information that you got from #(1)-#(4) on the grid.

	-3		-1		0		1		2
f'		$-$	0	+		+	0	$-$	
f''		+	+	+	0	$-$	$-$	$-$	
f	64	$\searrow$	-16	$\nearrow$	-8	$\nearrow$	0	$\searrow$	-16





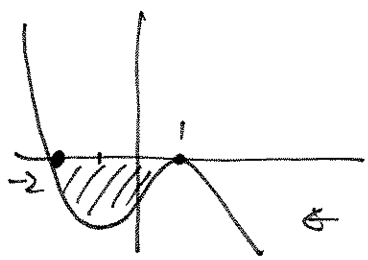
(6)(5 pts) Find the area surrounded by the curve $f(x) = -4x^3 + 12x - 8$ and the x-axis precisely. (Hint. Use the graph that you got from #(5) and factoring $-4x^3 + 12x - 8 = -4(x-1)^2(x+2)$ might be useful.)

We know the shape of the graph f(x) as 

So we just need to find x-intercepts :

$f(x)=0 \Rightarrow -4(x-1)^2(x+2)=0$

$\Rightarrow x = 1$ & -2.

So 

← below x-axis.

$$\text{Area}(\;) = -\int_{-2}^{1} f(x)\,dx$$

$$= -\int_{-2}^{1} -4x^3+12x-8. \qquad (F(x) = -x^4+6x^2-8x)$$

$$= -\left(\left[-(1)^4+6\cdot(1)^2-8(1)\right]-\left[-(-2)^4+6(-2)^2-8(-2)\right]\right)$$

$$= -\left((-1+6-8)-(-16+24+16)\right)$$

$$= -(-3-24)$$

$$= 27$$

2.(each 2 pts) Decide whether the following statements are necessarily always true or not. (You don't have to justify your answer. Just answer 'T' for 'True' and 'F' for 'False' statement. No partial credit will be given.)

(1) If the average cost of producing 10 items is \$1.00 dollar per unit and the marginal cost of producing 10 items is \$1.50 dollar per unit, then the average cost is reduced at producing 11 items. ($a(q) < C'(q) \Rightarrow a$ is increasing) ___F___

(2) If both the average cost and the marginal cost of producing 10 items are \$1.00 dollar per unit, then the derivative of the average cost function is 0 at 10 items assuming that both functions are continuous. ___T___

(3) The definite integral of a continuous function between a and b represents the area surrounded by the curve and the x-axis between a and b. (depends on the graph) ___F___

(4) If a continuous function $y = f(x)$ has the global maximum at $x = 1$, then $f'(1) = 0$.

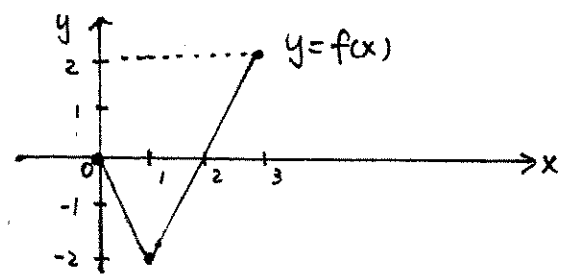
___F___ (cause global max can occur at one of the end points which is not a critical points.)

3.(6 pts) Find the following from the given graph of f.

(1) $\int_1^2 f(x)dx = -\text{area}$

$= -(2 \cdot 1 \cdot \frac{1}{2})$

$= -1$



(2) $\int_1^3 f(x)dx = \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx$

$= -\text{area} + \text{area}$

$= -(2 \cdot 1 \cdot \frac{1}{2}) + (2 \cdot 1 \cdot \frac{1}{2})$

$= -1 + 1 = 0$

4.(4 pts) The population in a town has the rate of change $f(t) = 10000\frac{1}{t}$ after t years since 2000 and the population in 2000 was exactly 10000. Find the population of the town in 2010 (that is, after 10 years since 2000).

The total change of the population between 2000 & 2010.

is $\int_0^{10} 10000 \cdot \frac{1}{t}\,dt = 10000(\ln 10) - 10000(\ln(0))$.

$F(t) = 10000 \ln t$

So the population in 2010 is

the population in 2000 + the total change

$$= \boxed{(10000) + \left(10000(\ln(10)) - 10000\,(\ln 0)\right)}$$

(Here, in fact, $\ln 0$ is $-\infty$

so we don't have to evaluate this!

In real life, it doesn't make sense,

but ~~in~~ mathematics, it happens! ^^)