

Name : Solution

Student ID # : _____

Honor 2201
Fall 2005
Instructor: Bo-Hae Im

EXAM 1
Friday, September 23, 2005

Problem	points	score
1	7	
2	6	
3	8	
4	6	
5	6	
6	7	
EC	3	
Total	40(+3)	

1-2. Let $f(x) = e^{2x} + 3 \cos x$ be a function.

1. (7 pts) Find the tangent line of $y = f(x)$ at $x = 0$.

$$\text{Point} = (0, f(0)) = (0, e^0 + 3 \cos 0) = (0, 1 + 3(1)) = \boxed{(0, 4)}$$

$$f'(x) = e^{2x} (2x)' + 3 (\cos x)'$$

$$= e^{2x} \cdot 2 + 3 (-\sin x)$$

$$= 2e^{2x} - 3 \sin x.$$

$$\Rightarrow \text{slope} = f'(0) = 2e^0 - 3 \sin 0 = 2 \cdot 1 - 3 \cdot 0 = \boxed{2}$$

So the equation of the tangent line at $x=0$ is

$$y - 4 = 2(x - 0).$$

$$\Rightarrow \boxed{y = 2x + 4}$$

2. (6 pts) Determine the concavity of the graph of $y = f(x)$ at $x = 0$. Justify your answer.

$$f''(x) = (f'(x))' = (2e^{2x} - 3 \sin x)'$$

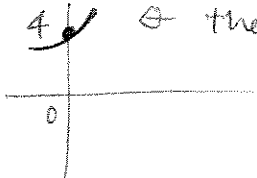
$$= 2 \cdot (e^{2x})' - 3 \cdot (\sin x)'$$

$$= 2 \cdot (2e^{2x}) - 3 (\cos x)$$

$$= 4e^{2x} - 3 \cos x$$

$$f''(0) = 4 \cdot e^0 - 3 \cdot \cos 0 = 4 \cdot 1 - 3 \cdot 1 = 4 - 3 = \boxed{1} > 0.$$

Since $f''(0) > 0$, $\boxed{f \text{ is concave up at } x=0.}$

( \oplus the graph around 0 will be like this!)

3. (each 2 pts) Decide whether the following statements are necessarily always true or not. (You don't have to justify your answer. Just answer 'T' for 'True' and 'F' for 'False' statement. No partial credit will be given.)

(1) If a function f satisfies that $f'(x) > 0$ for all x , then $f(x) > 0$ for all x as well.

(If $f(x)=x$, $f'(x)=1 > 0$ but \neq so $f(x) < 0$ for $x < 0$.)

F

(2) If $f'(x) = 0$ for all x , then $f''(x) = 0$ for all x .

T

(3) If $f''(x) < 0$ for all x , then the graph of $y = f'(x)$ is concave down for all x .

(For example, $f'(x) = -x$ & $f''(x) = -1 < 0$. but f' is not C.D.)

F

(4) If $f'(x)$ is a constant for all x , then $f''(x) = 0$ for all x .

T

4. (6 pts) The population in a town is 3000 at $t = 0$ and increases by 2% each year. Find the doubling time of the population in this town with specifying the unit.

$$P(t) = 3000 \cdot a^t \quad \text{with} \quad a = 1 + 0.02 = \underline{1.02}$$

$$= 3000 \cdot (1.02)^t$$

Doubling time t satisfies $2P_0 = P_0 \cdot a^t$

$$2 \cdot 3000 = 3000 \cdot (1.02)^t$$

$$2 = (1.02)^t$$

$$\ln 2 = \ln(1.02)^t$$

$$\ln 2 = t \cdot \ln(1.02)$$

\Rightarrow

$$t = \frac{\ln(2)}{\ln(1.02)} \quad \underline{\underline{\text{years}}}$$

(After $t = \frac{\ln 2}{\ln(1.02)}$ years, the population becomes
 $2 \cdot 3000 = 6000$.)

5.(6 pts) Find the quadratic polynomial $g(x) = ax^2 + bx + c$ which best fits $f(x) = 2 \ln x$ at $x = 1$ in the sense that

$$g(1) = f(1), \quad g'(1) = f'(1), \quad \text{and} \quad g''(1) = f''(1).$$

$$\begin{array}{l|l} f(x) = 2 \ln x & g(x) = ax^2 + bx + c \\ f'(x) = \frac{2}{x} & g'(x) = 2ax + b \\ f''(x) = -\frac{2}{x^2} & g''(x) = 2a. \end{array}$$

$$\begin{array}{l} g(1) = f(1) \Leftrightarrow a \cdot 1^2 + b \cdot 1 + c = 2 \ln 1 \Leftrightarrow a + b + c = 0 \\ g'(1) = f'(1) \Leftrightarrow 2a \cdot 1 + b = \frac{2}{1} \Leftrightarrow 2a + b = 2 \\ g''(1) = f''(1) \Leftrightarrow 2a = -\frac{2}{1^2} \Leftrightarrow 2a = -2. \end{array} \Leftrightarrow \left. \begin{array}{l} a + b + c = 0 \\ 2a + b = 2 \\ 2a = -2 \end{array} \right\}$$

Solve the system of linear equations

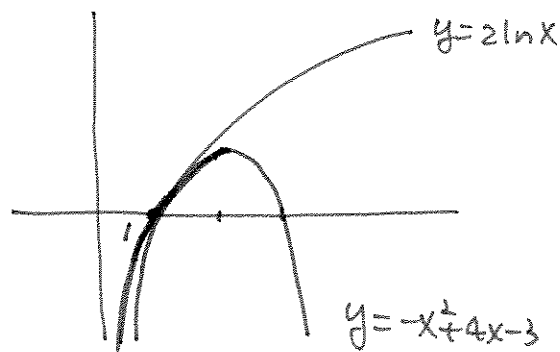
$$\begin{cases} a + b + c = 0 & \text{--- ①} \\ 2a + b = 2 & \text{--- ②} \\ 2a = -2 & \text{--- ③} \end{cases}$$

$$\Rightarrow a = -1 \quad \text{from ③}$$

$$\Rightarrow b = 4 \quad \text{from ②}$$

$$\Rightarrow c = -3 \quad \text{from ①}$$

So $\boxed{g(x) = -x^2 + 4x - 3}$ fits $f(x) = 2 \ln x$ the best at $x=1$.



6.(7 pts) Suppose that for a company to produce 30 machines, the total cost is \$2,000, the marginal cost \$24, the revenue \$2,300, and the marginal revenue \$25. Estimate the profit of producing 32 items for this company.

Find approximations of $C(32)$ & $R(32)$.

$$\text{Then } \pi(32) = R(32) - C(32).$$

$$\begin{aligned} \text{So } C(32) &\approx C'(30)(32-30) + C(30) \\ &= 24(2) + 2000 \\ &= \underline{\$2048} \end{aligned}$$

$$\begin{aligned} R(32) &\approx R'(30)(32-30) + R(30) \\ &= 25(2) + 2300 \\ &= \$2350. \end{aligned}$$

$$\text{So } \pi(32) = C(32) - R(32) \approx -2048 + 2350 = \$302.$$

$$\boxed{\pi(32) \approx \$302}.$$

(Extra-credit problem, 3 pts, no partial credit)

Estimate $e^{0.1}$. (Hint: You can use the fact that $e^0 = 1$.)

Let $f(x) = e^x$. Then $f(0) = e^0 = 1$ & $f'(0) = e^0 = 1$.

$$\begin{aligned} \& e^{0.1} = f(0.1) \approx f'(0)(0.1-0) + f(0) \\ &= 1(0.1) + 1 \\ &= 1.1 \end{aligned}$$

$$\text{So } \boxed{e^{0.1} \approx 1.1}$$