Solution key to Problems on Ch. 4

#1 & #2 are discussed in class. Refer to the class notes.

#3. (1) $f'(x) = 3x^2 - 6x - 9$, $\quad f''(x) = 6x - 6$.

(2) $3x^2 - 6x - 9 = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0 \Rightarrow$ $\boxed{x = 3, -1 \text{ are critical points.}}$

(3) $6x - 6 = 0 \Rightarrow x = 1$.

x		1	
f''	−	0	+

changes concavity. $\Rightarrow$ $\boxed{x = 1 \text{ is an inflection point.}}$

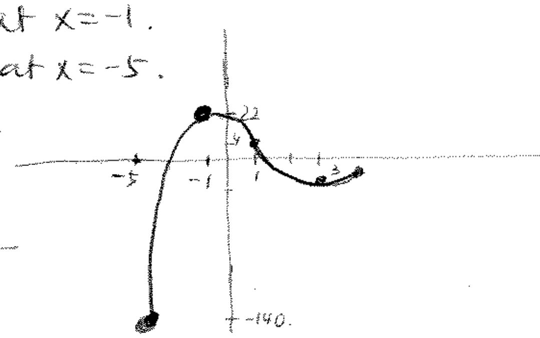
(4) $f''(3) > 0$. & $f''(-1) < 0$. $\Rightarrow$ f has a local min -13 at $x = 3$
& has a local max 22 at $x = -1$.

(5) $f(3) = -13$, $f(-1) = 22$, $f(-5) = -140$, $f(4) = -5$

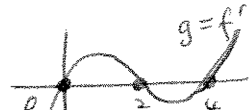
f has the global max 22 at $x = -1$.
& the global min -140 at $x = -5$.

(6)

x	-5		-1		1		3		4
f'	+	+	0	−	−	−	0	+	+
f''	−	−	−	−	0	+	+	+	+
f	-140	↗	22	↘	4	↘	-13	↗	-5



#4. Let $g(x) = f'(x) = x^3 - 6x^2 + 8x$, $0 \le x \le 5$. & Graph g $(= f')$.
Here $g(x) = x(x^2 - 6x + 8) = x(x-2)(x-4)$. So g is a cubic polynomial with vertical intercepts $x = 0, 2$ & 4. & the standard shape of a cubic polynomial is .

So the graph of g is  roughly!

f is increasing $\Leftrightarrow f' > 0 \Leftrightarrow g > 0 \Leftrightarrow$ $\underline{0 < x < 2 \text{ or } x > 4}$

f is decreasing $\Leftrightarrow f' < 0 \Leftrightarrow g < 0 \Leftrightarrow$ $\underline{x < 0 \text{ or } 2 < x < 4}$.

Critical points of f are such that $f' = 0$. $\Rightarrow f' = 0 \Leftrightarrow \underline{x = 0, 2, 4}$

$f''(x) = \underline{3x^2 - 12x + 8}$. & $f''(0) = 8 > 0$, $f''(2) = -4 < 0$, $f''(4) = 8 > 0$.

So f has local min. at $x = 0$ & $x = 4$.
& local max at $x = 2$.

No Inflection points! (even though $f'' = 0$ has a solution.)

#5. First, $(2,5)$ is satisfied $f(x) = a(x - b\ln x)$

$\Rightarrow \quad 5 = a(2 - b\ln 2)$. — ①

2 is a critical point. $\Rightarrow \quad f'(2) = 0$

$\Rightarrow f'(x) = a\left(1 - b\frac{1}{x}\right) \Rightarrow f'(2) = a\left(1 - \frac{b}{2}\right) = 0.$

$\Rightarrow \quad a = 0 \quad$ or $\quad 1 - \frac{b}{2} = 0 \quad \Rightarrow \quad a = 0$ or $b = 2$. — ②

If $a = 0$, then ① is not true. So $a \neq 0$. $\Rightarrow \boxed{b = 2}$

$\Rightarrow \quad 5 = a(2 - 2\cdot\ln 2) \quad \Rightarrow \quad \boxed{a = \frac{5}{2 - 2\ln 2}}$

We need to check $(2,5)$ is a local min.

$f''(2) = \frac{ab}{\cancel{x}^2} = \frac{ab}{4} = \frac{\left(\frac{5}{2 - 2\ln 2}\right)\cdot 2}{4} > 0$, since $\ln 2 < 1$.

$\Rightarrow$  local min at $(2,5)$. So $\boxed{a = \frac{5}{2 - 2\ln 2} \ \& \ b = 2}$

#6. $a(q) = b + mq$.

By definition, $a(q) = \frac{C(q)}{q}$. $\Rightarrow C(q) = a(q)\cdot q$.

$\Rightarrow \quad C'(q) = a'(q)\cdot q + a(q)\cdot 1$

$= (m)\cdot q + (b + mq)\cdot 1$

$= mq + b + mq$

$= \boxed{b + 2mq}$ //