HW Set 5 Solution (Partial)

(2) $f(x) = 2\sin x - x$ on $[0, \frac{\pi}{2}]$.

$f'(x) = 2\cos x - 1 = 0 \Rightarrow \cos x = \frac{1}{2} \Rightarrow \underline{x = \frac{\pi}{3}}$, $f''(x) = -2\sin x = 0 \Rightarrow \underline{x = 0}$.

	0		$\frac{\pi}{3}$		$\frac{\pi}{2}$
f'	$+$	$+$	0	$-$	$-$
f''	0	$-$	$-$	$-$	$-$
f	end pt.	⌒	local max	⌒	end pt.

$f(0) = 2\cdot 0 - 0 = 0$ } end points

$f(\frac{\pi}{2}) = 2\cdot 1 - \frac{\pi}{2} = 2 - \frac{\pi}{2}$

$f(\frac{\pi}{3}) = 2\cdot\frac{\sqrt{3}}{2} - \frac{\pi}{3} = \sqrt{3} - \frac{\pi}{3}$ ← local max.

(a) Critical point is $(\frac{\pi}{3}, \sqrt{3} - \frac{\pi}{3})$.

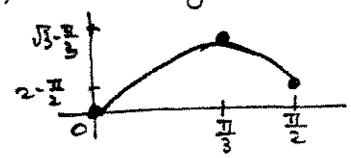
(b) no inflection point, cause 0 is one of the end point.

(c) f is increasing if $0 \le x < \frac{\pi}{3}$ & f is decreasing if $\frac{\pi}{3} < x \le \frac{\pi}{2}$.

(d) f has a local max $\sqrt{3} - \frac{\pi}{3}$ at $x = \frac{\pi}{3}$. & no local min.

(e) f has the global max $\sqrt{3} - \frac{\pi}{3}$ at $x = \frac{\pi}{3}$ & the global min 0 at $x = 0$.

(f)



(4) $f'(x) = e^x(x^2 - 2x + 1) + e^x(2x - 2) = e^x(x^2 - 2x + 1 + 2x - 2) = \underline{e^x(x^2 - 1)} = e^x(x+1)(x-1)$.

$\Rightarrow \underline{x = 1, -1}$. $f''(x) = e^x(x^2 - 1) + e^x(2x) = e^x(x^2 + 2x - 1) = e^x(x - (-1+\sqrt{2}))(x - (-1-\sqrt{2}))$.

$x = -1 + \sqrt{2}$, $\cancel{-1-\sqrt{2}}$. & $\sqrt{2} \approx 1.4142$. So $-1 - \sqrt{2} < -2$.

	$\cancel{-1-\sqrt{2}}$	-2		-1		$-1+\sqrt{2}$		1		3
f'			$+$	0	$-$	$-$	$-$	0	$+$	
f''			$-$	$-$	$-$	0	$+$	$+$	$+$	
f			↗	local max	⌒	inflection pt.	∪	local min	∪	

$f(-2) = e^{-2}(9) = \frac{9}{e^2}$ }

$f(3) = e^3 \cdot 4 = 4\cdot e^3$ }

$f(-1) = e^{-1}(4) = \frac{4}{e}$ ← local max

$f(1) = e \cdot 0 = 0$ ← local min

$f(-1+\sqrt{2}) = e^{-1+\sqrt{2}}(6 - 4\sqrt{2})$ ← Inflection.

(a) Critical points are $(-1, \frac{4}{e})$, $(1, 0)$

(b) Inflection points is $(-1+\sqrt{2}, e^{-1+\sqrt{2}}(6 - 4\sqrt{2}))$.

(c) f is increasing if $-2 \le x < -1$ & $1 < x \le 3$,
f is decreasing if $-1 < x < 1$.

(d) f has a local max $\frac{4}{e}$ at $x = -1$, local min 0 at $x = 1$.

(e) f has the global max $4\cdot e^3$ at $x = 3$, global min 0 at $x = 1$.

(f)