

Limbering Up

Two exercises to prepare us for the rigors of the journey ahead.

1. We conveniently finessed the problem of finding the equation of the straight line through two given points $P = (x_0, y_0)$ and $Q = (x_1, y_1)$, so that it could be left as an interesting exercise.

(a) We are looking for an equation of the form

$$y = mx + b,$$

where m and b are presently unknown. What we do know is that P and Q lie on this line, which means that we have the two equations

$$\begin{aligned} y_0 &= mx_0 + b \\ y_1 &= mx_1 + b. \end{aligned}$$

Solve these two equations for m and b , to obtain the desired equation. [Answers at the bottom of page 9.] (Remember that x_0, x_1, y_0 and y_1 are simply four known numbers—so this is just a simple problem involving two equations in the two unknowns m and b .)

- (b) Instead of this straightforward, unimaginative, approach, with its not very easily remembered result, we'll now try for a more elegant solution.

Answer the following two questions:

- (i) What must the slope m be?
- (ii) Now apply the *point-slope* form, using this value of m and the fact that the line goes through P , to obtain the desired equation.

- (c) Of course we could also use the point-slope form together with the fact that the line goes through Q . Check that the equation you obtain in this way agrees with that obtained in part (b).

2. This second exercise is basically just a joke, though it's the sort of joke that only a mathematician would find amusing. Nevertheless, it has a serious point, and there's a reason why it follows the previous exercise.

- (a) You are in a room with a gas stove, one that is working, and already lit, together with a table, and a pot of water on the table. You need some boiling water. What do you do? This part isn't meant to be tricky—just give the obvious answer

- (b) Now you are in the same room, with the same gas stove, still working, and already lit, together with the same table, and the same pot of water, but the pot of water is on the floor. You need some boiling water. What do you do? This part is meant to be tricky—you're supposed to give an "elegant" answer rather than the obvious one.

Problem 2(a). Put the pot of water on the stove.
 2(b). Put the pot of water on the table. (Because now you've reduced the problem to one that you already know how to solve, in just the same way that Exercise 1 reduced the problem of finding the equation of a line through two points to the point-slope form, which is the answer to a different problem.)

which is found to be equivalent, after multiplying everything out.

$$y - y_1 = \frac{y_0 - y_1}{x_0 - x_1}(x - x_1),$$

I(c). We also get

$$y - y_0 = \frac{y_1 - y_0}{x_1 - x_0}(x - x_0).$$

point-slope form gives reasoning as on page 7 [and as we computed in part (a)]. Then the I(b). The slope m clearly must be $y_1 - y_0/x_1 - x_0$, using the same by x_1 and the second by x_0 and then subtracting, gives a formula for b . Multiplying the first equation steadily eliminates b , giving a formula for m . Subtracting the second equation from the first eliminates