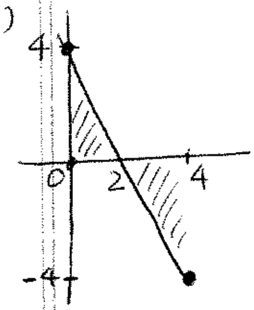
HW Set 9 Solution

#1. (1)



(2) Area = + = $2\cdot 4\cdot\frac{1}{2} + 2\cdot 4\cdot \frac{1}{2} = 8$

(3) (a) $\int_0^2 f(x)\,dx$ = Area of = 4

(b) $\int_2^4 f(x)\,dx$ = − Area of = $-2\cdot 4\cdot\frac{1}{2} = -4$

(c) $\int_0^4 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx = 4-4 = 0$

#2. (1) $\int_0^4 (-2x+4)\,dx = \Big[\underbrace{-x^2+4x}_{F(x)}\Big]_0^4 = (-4^2+4\cdot 4)-(-0^2+4\cdot 0) = \boxed{0}$

(2) $\int_{-1}^1 (x^3-x)\,dx = \Big[\underbrace{\frac{1}{4}x^4-\frac{1}{2}x^2}_{F(x)}\Big]_{-1}^1 = \left(\frac{1}{4}\cdot 1^4-\frac{1}{2}\cdot 1^2\right)-\left(\frac{1}{4}(-1)^4-\frac{1}{2}(-1)^2\right) = \boxed{0}$

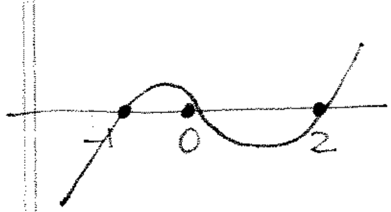
(3) $\int_{1}^{3} \frac{1}{x^2}\,dx = \int_1^3 x^{-2}\,dx = \left[\frac{x^{-1}}{-1}\right]_1^3 = \left[-\frac{1}{x}\right]_1^3 = -\frac{1}{3}-\left(-\frac{1}{1}\right) = \boxed{\frac{2}{3}}$.

#3. (1) (a) $\int_{-1}^0 [x^3-x^2-2x]\,dx = \Big[\underbrace{\frac{1}{4}x^4-\frac{1}{3}x^3-x^2}_{F(x)}\Big]_{-1}^0 = (0)-\left[\frac{1}{4}(-1)^4-\frac{1}{3}(-1)^3-(-1)^2\right] = \boxed{\frac{13}{12}}$

(b) $\int_0^2 [x^3-x^2-2x]\,dx \overset{①}{=} F(2)-F(0) = \left(\frac{1}{4}\cdot 2^4-\frac{1}{3}\cdot 2^3-2^2\right)-(0) = \boxed{-\frac{4}{3}}$

(c) $\int_{-1}^2 f(x)\,dx = \int_{-1}^0 f(x)\,dx + \int_0^2 f(x)\,dx = \frac{13}{12}-\frac{4}{3} = \frac{-3}{12} = \boxed{-\frac{1}{4}}$

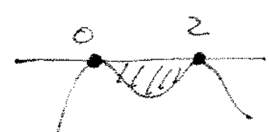
(2) $f(x) = x^3-x^2-2x = x(x^2-x-2) = x(x-2)(x+1) \Rightarrow$ x-intercepts $= 0, 2, -1$.



(3) Area = $_{-1}$ $_0$ + $_0$ $_2$ $= \int_{-1}^0 f(x)\,dx + \left(-\int_{-1}^2 f(x)\,dx\right) \overset{\text{by \#(1)}}{=} \frac{13}{12} + \left(-\left(-\frac{4}{3}\right)\right)$

$= \boxed{\frac{29}{12}}$

#4. $f(x) = -x^2(x^2-4x+4) = -x^2(x-2)^2 \Rightarrow$ x-intercepts $= 0, 2$.



Area = $_0$ $_2$ $= -\int_0^2 f(x)\,dx = -\int_0^2 (-x^4+4x^3-4x^2)\,dx$

$= -\left[-\frac{1}{5}x^5+x^4-\frac{4}{3}x^3\right]_0^2 = -\left(-\frac{1}{5}\cdot 2^5+2^4-\frac{4}{3}\cdot 2^3\right) = \boxed{\frac{16}{15}}$