HW Set 6 Solution.

#1. Find c in $(-2,2)$ such that $f'(c) = \frac{f(2)-f(-2)}{2-(-2)}$. Here $f'(c) = 2c+2$.

So solve $2c+2 = \frac{(2^2+2)-((-2)^2+(-2))}{2-(-2)}$ for c. $\Rightarrow$ $\boxed{c = -\frac{1}{2}}$ in $(-2,2)$.

#2. Find c in $(1,3)$ such that $f'(c) = \frac{f(3)-f(1)}{3-1}$. Here $f'(c) = -\frac{1}{c^2}$.

So solve $-\frac{1}{c^2} = \frac{(\frac{1}{3}+1)-(\frac{1}{1}+1)}{3-1}$ for c. $\Rightarrow$ $c^2=3 \Rightarrow c = \pm\sqrt{3}$.

But we need to choose $\boxed{c=\sqrt{3}}$ only, cause it must be in $(1,3)$.

#3. Let $f(x) = x - \cos x$. Then $f(0) = 0 - \cos 0 = -1 < 0$ and $f(\frac{\pi}{2}) = \frac{\pi}{2} - \cos\frac{\pi}{2} = \frac{\pi}{2} > 0$.

So $f(0) < 0 < f(\frac{\pi}{2})$. Then $\boxed{\text{by Intermediate Value Theorem (IVT)}}$,

$f(x)=0$ has at least one solution c, that is, $f(c)=0$ in $(0,\frac{\pi}{2})$. ∎

#4. $f'(x) = -\frac{2}{x^2} < 0$ for all x in $(-2,-1)$.

So by theorem, f must be increasing on $(-2,-1)$.

#5. Let $h(x) = f(x) - g(x)$. on $[1,5]$. Then $h'(x) = f'(x) - g'(x) = 0$ by assumption.

So $h'(x) = 0$. Then $h(x) = C$ (constant) on $[1,5]$.

We need to determine C. Since $h(2) = f(2) - g(2) = 1 - 4 = -3$, $\boxed{h(x) = -3}$.

Hence, $f(3) - g(3) = h(3) = \boxed{-3}$ and $f(4) - g(4) = h(4) = \boxed{-3}$.

#6. Since $f'(x) = 0$ on $(1,4)$, $f(x) = C$ (constant) on $(1,4)$.

Since $f(2) = 4$, $f(x) = 4$ on $[1,4]$. $\Rightarrow$ $\boxed{f(x) = 4 \text{ on } [1,4]}$.

#7. (1) $\left\{\begin{array}{l}\text{Choose } 0 \,\&\, 1.\\ \&\text{ let } f(x) = 4x^3+2x-1\end{array}\right.$ Then $f(0) = -1 < 0$ & $f(1) = 4+2-1 = 5 > 0$.

So $f(0) < 0 < f(1)$.

Then by <u>IVT</u>, there exists at least one c in $(0,1)$ such that

$f(c) = 0$. $\Rightarrow$ c is a solution of $4x^3+2x-1=0$. ∎

(2). We have shown that $f(x)=0$ has at least one solution by #(1).

Suppose there are c_1 & c_2 as solutions of $f(x)=0$.

Then $f(c_1) = 0 = f(c_2)$. Then by $\boxed{\text{Rolle's theorem}}$, (or MVT),

there exists c between c_1 and c_2 such that $f'(c) = 0$.

But $f'(x) = 12x^2+2$ has no solution, that is, there is no x such that $12x^2+2=0$. So such c that $f'(c)=0$ cannot exist.

This contradiction happens because we assume that there are <u>two</u> solutions.

So $f(x)=0$ has only one solution!