HW Set 5 Solution

#1. (1) $f(x) = -x^3 + 3x^2 + 9x - 12$ on $[-2, 4]$.

$f'(x) = -3x^2 + 6x + 9 = -3(x^2 - 2x - 3) = -3(x-3)(x+1). = 0 \Rightarrow x = \underline{-1, 3}$.

$f''(x) = -6x + 6 = -6(x-1) = 0 \Rightarrow \underline{x = 1}$.

		-1		1		3	
f'	−	0	+	+	+	0	−
f''	+	+	+	0	−	−	−
f	◟	local min	◞	Inflection point	◜	local max	◝

$\begin{cases} f(1) = -1 & \leftarrow \text{Inflection point} \\ f(3) = 15 & \leftarrow \text{local max} \\ f(-1) = -17 & \leftarrow \text{local min} \end{cases}$

$\left.\begin{cases} f(-2) = -10 \\ f(4) = 0 \end{cases}\right\}$ end points

(a) Critical points are $(-1, -17)$ & $(3, 15)$.

(b) Inflection point is $(1, -1)$, since f changes concavity at $x = 1$.

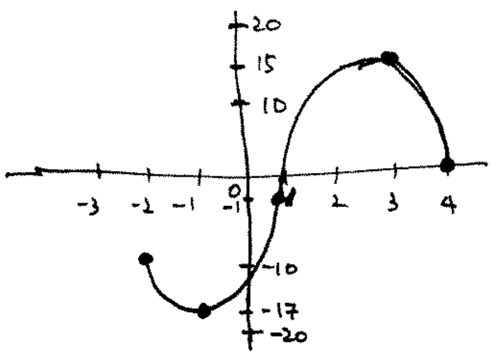
(c) f is increasing if $f' > 0$, that is, $\boxed{-1 < x < 1.}$

f is decreasing if $f' < 0$, that is $\boxed{-2 \le x < -1}$ and $\boxed{3 < x \le +4.}$

(d) f has $\underline{\text{a local max } 15}$ at $x=3$ & $\underline{\text{a local min } -17}$ at $x = -1$.

(e) f has $\underline{\text{the global max } 15}$ at $x = 3$ & $\underline{\text{the global min } -17}$ at $x = -1$.

(f)



(2) $f(x) = 2\sin x - x$ on $[0, \frac{\pi}{2}]$.

$f'(x) = 2\cos x - 1. = 0 \Rightarrow \cos x = \frac{1}{2} \Rightarrow \underline{x = \frac{\pi}{3}.}$, $f''(x) = -2\sin x = 0 \Rightarrow \underline{x = 0.}$

	0		$\frac{\pi}{3}$		$\frac{\pi}{2}$
f'	+	+	0	−	−
f''	0	−	−	−	−
f	end pt.	◜	local max.	◝	end pt.

$\left.\begin{array}{l} f(0) = 2\cdot 0 - 0 = 0 \\ f(\frac{\pi}{2}) = 2\cdot 1 - \frac{\pi}{2} = 2 - \frac{\pi}{2} \end{array}\right\}$ end points

$f(\frac{\pi}{3}) = 2\cdot\frac{\sqrt{3}}{2} - \frac{\pi}{3} = \sqrt{3} - \frac{\pi}{3}$ $\leftarrow$ local max.

(a) Critical point is $(\frac{\pi}{3}, \sqrt{3} - \frac{\pi}{3})$.

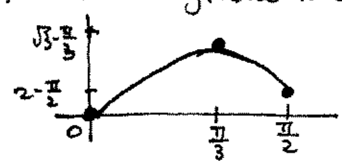
(b) no inflection point, cause 0 is one of the end point.

(c) f is increasing if $0 \le x < \frac{\pi}{3}$ & f is decreasing if $\frac{\pi}{3} < x \le \frac{\pi}{2}$.

(d) f has a local max $\sqrt{3} - \frac{\pi}{3}$ at $x = \frac{\pi}{3}$. & no local min.

(e) f has the global max $\sqrt{3} - \frac{\pi}{3}$ at $x = \frac{\pi}{3}$ & the global min 0 at $x = 0$.

(f)



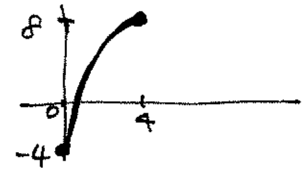
(3) $f'(x) = (6\sqrt{x}-4)' = (6\cdot x^{\frac{1}{2}}-4)' = 6\cdot\frac{1}{2}x^{-\frac{1}{2}} = 3\cdot\frac{1}{\sqrt{x}}$, $f''(x) = -\frac{3}{2}\cdot\frac{1}{\sqrt{x^3}}$

f' is undefined at $\underline{\underline{x=0}}$. , f'' is undefined at $x=0$.

	0		4
f'		$+$	
f''		$-$	
f		⌒	

$f(0) = 6\cdot 0-4 = -4$.

$f(4) = 6\cdot\sqrt{4}-4 = 6\cdot 2-4 = 8$,

(f) 

(a) critical point is $(0,-4)$.

(b) no inflection point, cause it is always concave down.

(c) f is increasing if $0\le x\le 4$.

(d) f has no local min / no local max.

(e) f has the global min -4 at $x=0$ & global max 8 at $x=4$.

(4) $f'(x) = e^x(x^2-2x+1)+e^x(2x-2) = e^x(x^2-2x+1+2x-2) = \underline{e^x(x^2-1)} = e^x(x+1)(x-1)$.

$\Rightarrow \underline{x=1,-1}$. $\quad f''(x) = e^x(x^2-1)+e^x(2x) = e^x(x^2+2x-1) = e^x(x-(-1+\sqrt{2}))(x-(-1-\sqrt{2}))$.

$x = -1+\sqrt{2}$, $\cancel{-1-\sqrt{2}}$. $\quad$ & $\sqrt{2}\approx 1.4142$. So $-1-\sqrt{2} < -2$.

	$\cancel{-1-\sqrt{2}}$	-2		-1		$-1+\sqrt{2}$		1		3
f'			$+$	0	$-$	$-$	$-$	0	$+$	
f''			$-$	$-$	$-$	0	$+$	$+$	$+$	
f				local max		inflection pt.		local min		

$f(-2) = e^{-2}\cdot(9) = \frac{9}{e^2}$

$f(3) = e^3\cdot 4 = 4\cdot e^3$

$f(-1) = e^{-1}(4) = \frac{4}{e}$ ← local max

$f(1) = e\cdot 0 = 0$ ← local min

$f(-1+\sqrt{2}) = e^{-1+\sqrt{2}}(6-4\sqrt{2})$ ← Inflection.

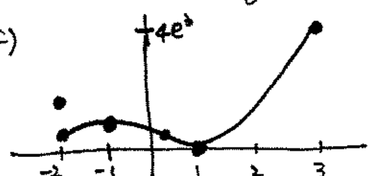
(a) Critical points are $(-1,\frac{4}{e})$, $(1,0)$

(b) Inflection point is $(-1+\sqrt{2},\ e^{-1+\sqrt{2}}(6-4\sqrt{2}))$.

(c) f is increasing if $-2\le x<-1$ & $1<x\le 3$,
f is decreasing if $-1<x<1$.

(d) f has a local max $\frac{4}{e}$ at $x=-1$, local min 0 at $x=1$.

(e) f has the global max $4\cdot e^3$ at $x=3$, global min 0 at $x=1$.

(f) 

(5) $f'(x) = \ln(x-1)+(x-1)\cdot\frac{1}{(x-1)} = \ln(x-1)+1 = 0 \Rightarrow x-1 = e^{-1} \Rightarrow \underline{x = 1+\frac{1}{e}}$

$f''(x) = \frac{1}{x-1} \Rightarrow x$ is undefined at $x=1$.

	1		$1+\frac{1}{e}$	2
f'		$-$	0	$+$
f''		$+$	$+$	$+$
f			local min	

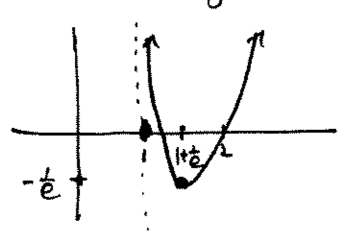
(a) Critical point is $(1+\frac{1}{e},\ (1+\frac{1}{e}-1)\cdot\ln(1+\frac{1}{e}-1)) = (1+\frac{1}{e}, -\frac{1}{e})$.

(b) Inflection point does not exist.

(c) f is increasing if $x>1+\frac{1}{e}$ & f is decreasing if $1<x<1+\frac{1}{e}$.

(d) f has no local max & a local min $-\frac{1}{e}$ at $x=1+\frac{1}{e}$.

(d) f has the global min $-\frac{1}{e}$ at $x=1+\frac{1}{e}$,
& no global max (cause max $=\infty$.)

(f) 

#2. Let (x, y) be a point on $y^2 = 4x$[redacted].

Let f be the square of the distance between (x,y) & $(2,0)$.

Then $f = (x-2)^2 + (y-0)^2 = (x-2)^2 + y^2 = (x-2)^2 + (4x$[redacted]$)$[redacted]

$\Rightarrow f(x) = (x-2)^2 + 4x$[redacted]. $\Rightarrow f'(x) = 2(x-2) +$ [redacted] $= 0 \Rightarrow x = 0.$

$f''(x) = 2. > 0.$ $\qquad = 2(32x^3 + x - 2).$

So f has a local min at $x=0$. local & global min.

If $x = 0$, then the square of the distance from $(2,0)$

has the minimum. $\sqrt{(0-2)^2 + 4\cdot 0} = \sqrt{4+0} = 2.$

& the point on $y^2 = 4x$ at $x=0$ is $(0, 0)$.

So the point $\boxed{(0,0)}$ minimizes the distance from $(2,0)$

and the minimum distance is 2.

#3. $f'(x) = 0 \Rightarrow x = 1, -2.$

$f''(x) = -(x+2)^2 - (x-1)\cdot 2(x+2) = -(x+2)(x+2+2(x-1))$

$= -(x+2)(3x) \Rightarrow x = 0, -2.$

		-2		0		1	
f'	$+$	0	$+$	$+$	$+$	0	$-$
f''	[redacted]	[redacted]	$+$	0	$-$	$-$	$-$
f		inflection, no local		inflection		local max.	

f has a local max at $x=1$, and no local min.

A rough graph of f will be