HW 4 Solution (1080)

#1. (1) $f'(x) = \frac{1}{2x^2+1} \cdot (2x^2+1)' = \boxed{\frac{4x}{2x^2+1}}$

(2) $f'(x) = \frac{1}{\cos x} \cdot (\cos x)' = \boxed{\frac{-\sin x}{\cos x}} \left(\underset{or}{=} -\tan x \right)$

(3) $f'(x) = e^{\tan x} \cdot (\tan x)' = e^{\tan x} \cdot \frac{1}{\cos^2 x} = \boxed{\frac{e^{\tan x}}{\cos^2 x}}$

(4) $f'(x) = 2 \cdot (\ln x)^{2-1} \cdot (\ln x)' = 2 \cdot \ln x \cdot \frac{1}{x} = \boxed{\frac{2 \ln x}{x}}$

(5) $f'(x) = \frac{1}{x^2} \cdot (x^2)' = \frac{2x}{x^2} = \boxed{\frac{2}{x}}$.

#2.

$\log_{10} 5 + \log_{10} 20 - 4 \ln(\sqrt{e}) + 15 \cdot 2^{\log_2(\frac{1}{5})} - e^{\ln 2} + \log_{18} 270 - \log_{18} 15$

$= \log_{10}(5 \cdot 20) - 4 \ln(e^{\frac{1}{2}}) + 15 \cdot \frac{1}{5} - 2 + \log_{18}\left(\frac{270}{15}\right)$

$= \log_{10}(100) - 4 \cdot \frac{1}{2} \cdot \ln e + 3 - 2 + \log_{18} 18$

$= \log_{10} 10^2 - 2 \cdot 1 + 3 - 2 + 1 = 2 - 2 + 3 - 2 + 1 = \boxed{2}$.

#3. (1) Let $f(x) = x^{10}$. $f'(x) = 10x^9$, $f''(x) = 90 \cdot x^8$.

Let $a = 2$. Then $(2.01)^{10} = f(2.01) \approx f(a) + f'(a)(2.01 - a)$

$= 2^{10} + 10 \cdot 2^9 (2.01 - 2)$

$= 1024 + 5120 \cdot (0.01) = \boxed{1075.20}$

(2) $(2.01)^2 = f(2.01) \approx f(a) + f'(a)(2.01 - a) + f''(a)(2.01 - a)^2$

$= \underbrace{2^{10} + 10 \cdot 2^9 (2.01 - 2)} + 90 \cdot 2^8 \cdot (2.01 - 2)^2$

$= 1075.20 + 23040 \cdot (0.01)^2 = 1075.20 + 2.3040 = \boxed{1077.5040}$

(3) Difference between 1075.20 & 1076.367 is 1.167 & between 1077.5040 & 1076.367 is 1.137. So 2nd order approximation is better.

#4. $s(t) = t^3 - 9t^2 + 24t$, $v(t) = 3t^2 - 18t + 24 = 3(t-2)(t-4)$, $a(t) = 6t - 18 = 6(t-3)$.

(1) moving direction changes when $v(t) = 0$. So solve $v(t) = 0$ for t. $\Rightarrow 3(t-2)(t-4) = 0 \Rightarrow \boxed{t = 2,\ t = 4}$

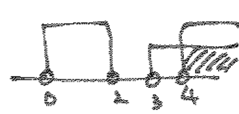
(2) Moving forwards means $v(t) > 0$. So solve $v(t) > 0$. $\Rightarrow \boxed{0 < t < 2 \text{ or } 4 < t}$.

(3) moving backwards means $v(t) < 0$. Solve $v(t) < 0 \Rightarrow \boxed{2 < t < 4}$

(4) moving forwards faster & faster means that $s(t)$ looks like CU.

So solve $v(t) > 0$ and $a(t) > 0$.

$\Rightarrow 3(t-2)(t-4) > 0$ and $6(t-3) > 0$

$\Rightarrow 0 < t < 2$ or $4 < t$ and $t > 3$ ( overlap.)

$\Rightarrow \boxed{4 < t}$