HW Set 2 Solution

#2. (1) $\begin{pmatrix}\text{Average rate of change}\\ \text{between } 1 \,\&\, 2\end{pmatrix} = \dfrac{S(2)-S(1)}{2-1} = \dfrac{(-2^2+2\cdot 2+1)-(-1^2+2\cdot 1+1)}{1} = -1$

$\begin{pmatrix}\text{Average rate of change}\\ \text{between } 2 \,\&\, 3\end{pmatrix} = \dfrac{S(3)-S(2)}{3-2} = \dfrac{(-3^2+2\cdot 3+1)-(-2^2+2\cdot 2+1)}{1} = -3$

(2) Since both velocity is negative, it moves backwards.
But between 1 & 2, the speed is $|-1|=1$ & between 2 & 3, the speed is $|-3|=3$. So it speeds up. So it moves faster & faster backwards.

#3. (1) At $x=1$, $f(1)=0$ exists.

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \dfrac{x^2-2x+1}{x-1} = \lim_{x\to 1^+} \dfrac{(x-1)^2}{(x-1)} = \lim_{x\to 1^+} \dfrac{(x-1)}{1} = 1-1 = 0$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \dfrac{x^2-2x+1}{x-1} = \lim_{x\to 1^-} \dfrac{(x-1)^2}{x-1} = \lim_{x\to 1^-} \dfrac{(x-1)}{1} = 1-1 = 0$

equal

So $\lim_{x\to 1} f(x) = 0$ exists.

Moreover, $f(1) = 0 = \lim_{x\to 1} f(x)$, So f is continuous at $x=1$.

Then for any $x\neq 1$, $f(x) = \dfrac{x^2-2x+1}{x-1}$ is well-defined & continuous.

So f is continuous everywhere.

(2) At $x=0$, $f(0)=0$ exists.

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \dfrac{x}{|x|} = \lim_{x\to 0^+} \dfrac{x}{x} = \lim_{x\to 0^+} 1 = 1$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \dfrac{x}{|x|} = \lim_{x\to 0^-} \dfrac{x}{-x} = \lim_{x\to 0^-} -1 = -1$

not equal.

So $\lim_{x\to 0} f(x)$ does not exist. $\Rightarrow$ f is NOT continuous at $x=0$.

But for any $x\neq 0$, $f(x) = \dfrac{x}{|x|} = \begin{cases} 1 \\ -1 \end{cases}$ is well-defined & continuous.

So f is continuous for all $x\neq 0$. (But not everywhere.)

(3) At $x=0$, $f(0)=0$ exists.

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \dfrac{x^2-3x}{x} = \lim_{x\to 0^+} \dfrac{x(x-3)}{x} = \lim_{x\to 0^+} \dfrac{(x-3)}{1} = 0-3 = -3$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \dfrac{x^2-3x}{x} = \cdots = 0-3 = -3$

equal.

So $\lim_{x\to 0} f(x) = -3$ exists.

But $f(0)=0$ & $\lim_{x\to 0} f(x) = -3$ so they are not equal.

$\Rightarrow$ f is NOT continuous at $x=0$.

But for any $x\neq 0$, f is continuous.

So f is continuous for all $x\neq 0$. (But not everywhere).

#4. (1) $f'(t) = \lim\limits_{x\to t} \frac{f(x)-f(t)}{x-t} = \lim\limits_{x\to t} \frac{(3x+1)-(3t+1)}{x-t} = \lim\limits_{x\to t} \frac{3(x-t)}{(x-t)} = \lim\limits_{x\to t} 3 = \boxed{3}$.

So $\underline{f'(t)=3 \ \& \ f'(1)=3 \ \& \ f'(0)=3}$.

(2) $f'(t) = \lim\limits_{x\to t} \frac{f(x)-f(t)}{x-t} = \lim\limits_{x\to t} \frac{4-4}{x-t} = \lim\limits_{x\to t} 0 = \boxed{0}$.

So $\underline{f'(t)=0 \ \& \ f'(1)=0 \ \& \ f'(0)=0}$.

(3) $f'(t) = \lim\limits_{x\to t} \frac{f(x)-f(t)}{x-t} = \lim\limits_{x\to t} \frac{(2x^2-1)-(2t^2-1)}{x-t} = \lim\limits_{x\to t} \frac{2(x^2-t^2)}{x-t}$

$= \lim\limits_{x\to t} \frac{2(x-t)(x+t)}{(x-t)} = \lim\limits_{x\to t} \frac{2(x+t)}{1} = 2(t+t) = 2\cdot 2t = \underline{4t}$

So $\underline{f'(t)=4t \ \& \ f'(1)=4\cdot 1=4 \ \& \ f'(0)=4\cdot 0=0}$

(4) $f'(t) = \lim\limits_{x\to t} \frac{f(x)-f(t)}{x-t} = \lim\limits_{x\to t} \frac{(-x^3+2x-1)-(-t^3+2t-1)}{x-t} = \lim\limits_{x\to t} \frac{-(x^3-t^3)+2(x-t)}{(x-t)}$

$= \lim\limits_{x\to t} \frac{-(x-t)(x^2+xt+t^2)+2(x-t)}{(x-t)} = \lim\limits_{x\to t} \frac{(x-t)\left[-(x^2+xt+t^2)+2\right]}{(x-t)}$

$= \lim\limits_{x\to t} \left[-(x^2+xt+t^2)+2\right] = -(t^2+t\cdot t+t^2)+2$

$= \underline{-3t^2+2}$

So $\underline{f'(t) = -3t^2+2 \ \& \ f'(1) = -3\cdot 1^2+2 = -1 \ \& \ f'(0) = -3\cdot 0^2+2 = 2}$

(5) $f'(t) = \lim\limits_{x\to t} \frac{f(x)-f(t)}{x-t} = \lim\limits_{x\to t} \frac{\frac{1}{x}-\frac{1}{t}}{x-t} = \lim\limits_{x\to t} \frac{\frac{t-x}{xt}}{(x-t)} = \lim\limits_{x\to t} \frac{-(x-t)}{xt(x-t)}$

$= \lim\limits_{x\to t} \frac{-1}{xt} = \frac{-1}{t\cdot t} = -\frac{1}{t^2}$.

So $\underline{f'(t) = -\frac{1}{t^2} \ \& \ f'(1) = -\frac{1}{1^2} = -1 \ \& \ f'(0) = -\frac{1}{0}}$ is undefined!