HW 11. Solution

#1. (1) $\int_2^{e^3+1} \frac{\ln(x-1)}{(x-1)}\,dx = \int_0^3 u\,du \underset{F(u)=\frac{u^2}{2}}{=} F(3)-F(0) = \frac{3^2}{2}-\frac{0^2}{2} = \boxed{\frac{9}{2}}$

$$\begin{cases} u=\ln(x-1) \\ du=\frac{1}{x-1}\,dx \\ x=2 \Rightarrow u=\ln(2-1)=\ln 1=0 \\ x=e^3+1 \Rightarrow u=\ln(e^3+1-1)=\ln e^3=3 \end{cases}$$

(2) $\int_0^1 x(2x^2+1)^{20}\,dx = \int_1^3 u^{20}\cdot\frac{du}{4} = \frac{1}{4}\int_1^3 u^{20}\,du \underset{f(u)=\frac{u^{21}}{21}}{=} \frac{1}{4}(F(3)-F(1)) = \frac{1}{4}\left(\frac{3^{21}}{21}-\frac{1}{21}\right) = \boxed{\frac{3^{21}-1}{4\cdot 21}}$.

$$\begin{cases} u=2x^2+1 \\ du=4x\,dx \Rightarrow \frac{du}{4}=x\,dx \\ x=0 \Rightarrow u=2\cdot 0^2+1=1 \\ x=1 \Rightarrow u=2\cdot 1^2+1=3 \end{cases}$$

(3) $\int_0^{\frac{\pi}{6}} \cos^3 x\,dx \underset{(\cos^2 x=1-\sin^2 x)}{=} \int_0^{\frac{\pi}{6}} (1-\sin^2 x)\cos x\,dx = \int_0^{\frac{1}{2}} (1-u^2)\,du \underset{F(u)=u-\frac{u^3}{3}}{=} F(\tfrac{1}{2})-F(0) = \frac{11}{24}$

$$\begin{cases} u=\sin x \\ du=\cos x\,dx \\ x=0 \Rightarrow u=\sin 0=0 \\ x=\frac{\pi}{6} \Rightarrow u=\sin\frac{\pi}{6}=\frac{1}{2} \end{cases}$$

(4) $\int_{\frac{\pi}{3}}^{\pi} \sin^5 x\,dx \underset{(\sin^2 x=1-\cos^2 x)}{=} \int_{\frac{\pi}{3}}^{\pi} (1-\cos^2 x)^2\cdot\sin x\,dx = \int_{\frac{1}{2}}^{-1} (1-u^2)^2\cdot(-du) = -\int_{\frac{1}{2}}^{-1} 1-2u^2+u^4\,du$

$$\begin{cases} u=\cos x \\ du=-\sin x\,dx \Rightarrow -du=\sin x\,dx \\ x=\frac{\pi}{3} \Rightarrow u=\cos\frac{\pi}{3}=\frac{1}{2} \\ x=\pi \Rightarrow u=\cos\pi=-1 \end{cases}$$

$\underset{F(u)=u-2\frac{u^3}{3}+\frac{u^5}{5}}{=} -\left[F(-1)-F(\tfrac{1}{2})\right] = \boxed{\frac{153}{160}}$

(5) $\int_1^3 (x-1)\,e^{x^2-2x-1}\,dx = \int_{-2}^{2} e^u\,\frac{du}{2} = \frac{1}{2}\int_{-2}^{2} e^u\,du \underset{F(u)=e^u}{=} \boxed{\frac{1}{2}\left[e^2-e^{-2}\right]}$

$u=x^2-2x-1$
$du=(2x-2)\,dx$
$du=2(x-1)\,dx$
$\frac{du}{2}=(x-1)\,dx$

$x=1 \Rightarrow u=1-2-1=-2$
$x=3 \Rightarrow u=9-6-1=2$

#2. (1) $V(S_1) = \int_0^2 \pi f(x)^2\,dx \underset{\text{by Disk method.}}{=} \int_0^2 \pi(2x+1)^2\,dx = \boxed{\frac{62}{3}\pi}$

(2) $V(S_2) = \int_0^2 2\pi x\cdot f(x)\,dx \underset{\text{Shell method}}{=} 2\pi\int_0^2 x(2x+1)\,dx = \boxed{\frac{44}{3}\pi}$

(3) $V(S_1) = \int_1^4 \pi f(x)^2\,dx \underset{\text{by Disk method}}{=} \int_1^4 \pi(x^2-1)\,dx = \boxed{18}$

#2. (4) $V(S_2) = \int_1^4 2\pi x \cdot f(x)\,dx = \int_1^4 2\pi x \cdot \sqrt{x^2-1}\,dx = \int_0^{15} \pi \cdot \sqrt{u}\,du$

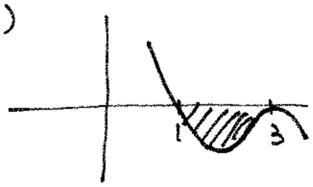
by ~~[scribble]~~ shell metho.

$$\begin{cases} u = x^2-1 \\ du = 2x\,dx \\ x=1 \Rightarrow u=0 \\ x=4 \Rightarrow u=15 \end{cases}$$

$= \pi \int_0^{15} u^{\frac{1}{2}}\,du = \pi\left(F(15) - F(0)\right) = \pi \cdot \dfrac{15^{\frac{3}{2}}}{\frac{3}{2}} = \boxed{\dfrac{2\pi}{3} \cdot 15\sqrt{15}}$ or $= \boxed{10\pi\sqrt{15}}$

$F(u) = \dfrac{u^{\frac{3}{2}}}{\frac{3}{2}}$

#3. (1)



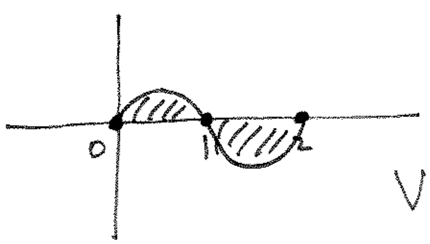
By shell method,

$V = \int_1^3 2\pi x \cdot (-f(x))\,dx = +\int_1^3 2\pi x\left(+(x-1)(x-3)^2\right)dx$

$= 2\pi \int_1^3 (x^2-x)(x^2-6x+9)\,dx$

$= 2\pi \int_1^3 x^4 - 7x^3 + 15x^2 - 9x\,dx = \boxed{\dfrac{24}{5}\pi}$

(2)



By shell method,

$V = \int_0^1 2\pi x f(x)\,dx + \int_1^2 2\pi x \cdot (-f(x))\,dx$

$= 2\pi \int_0^1 x \cdot x(x-1)(x-2)\,dx - 2\pi \int_1^2 x \cdot x(x-1)(x-2)\,dx$

$= 2\pi \int_0^1 (x^4 - 3x^3 + 2x^2)\,dx - 2\pi \int_1^2 x^4 - 3x^3 + 2x^2\,dx$

$= \boxed{3\pi}$