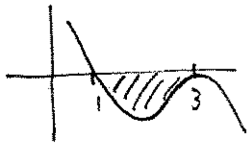
HW Set 10. Solution.

#1. (1) x-intercepts = 1, 3



$$f(x) = -(x-1)(x-3)^2 = -x^3+7x^2-15x+9$$

$$\text{Area} = -\int_1^3 (-x^3+7x^2-15x+9)\,dx = \int_1^3 (x^3-7x^2+15x-9)\,dx$$

$\left(F(x)=\frac{1}{4}x^4-\frac{7}{3}x^3+\frac{15}{2}x^2-9x\right)$ $\quad = F(3)-F(1) = \boxed{\frac{4}{3}}$

(2) x-intercepts = 1, −1

$$f(x) = (x-1)^3(x+1) = x^4-2x^3+2x-1$$

$$\text{Area} = -\int_{-1}^1 (x^4-2x^3+2x-1)\,dx = -(F(1)-F(-1)) = \boxed{\frac{8}{5}}$$

$\left[F(x)=\frac{1}{5}x^5-\frac{2}{4}x^4+x^2-x\right]$

#2. (1) $F(x)=\ln|x|+x+C$, $\quad 2=F(1)=\ln 1+1+C \Rightarrow C=1. \Rightarrow \boxed{F(x)=\ln x+x+1}$

(Here, we assumed that $x>0$ so $\ln|x|=\ln x$.)

(2) $F(x)=\frac{1}{4}x^4+x^3-x+C$, $\quad 1=F(0)=C \Rightarrow C=1 \Rightarrow \boxed{F(x)=\frac{1}{4}x^4+x^3-x+1}$

(3) $F(x)=\frac{1}{3}e^{3x-1}+x+C$, $\quad \frac{5}{3}=F(\frac{1}{3})=\frac{1}{3}e^0+\frac{1}{3}+C \Rightarrow C=1 \Rightarrow \boxed{F(x)=\frac{1}{3}e^{3x-1}+x+1}$

#3. (1)

$$\int_0^{\frac{\pi}{6}} \sin x\cos x\,dx = \int_0^{\frac{1}{2}} u\,du = \left[\frac{1}{2}u^2\right]_0^{\frac{1}{2}} = \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2-\frac{1}{2}(0)^2 = \boxed{\frac{1}{8}}$$

$\left[\begin{array}{l} u=\sin x \\ du=\cos x\,dx \\ x=0 \Rightarrow u=0 \\ x=\frac{\pi}{6} \Rightarrow u=\frac{1}{2} \end{array}\right]$

(2) $\left[\begin{array}{l} u=x-1 \\ du=dx \\ x=1 \Rightarrow u=0 \\ x=2 \Rightarrow u=1 \end{array}\right]$

$$\int_1^2 (x-1)^{20}\,dx = \int_0^1 u^{20}\,du = \left[\frac{1}{21}u^{21}\right]_0^1 = \frac{1}{21}(1)^{21}-\frac{1}{21}(0)^{21} = \boxed{\frac{1}{21}}$$

(3) $\left[\begin{array}{l} u=3x-1 \\ du=3\,dx \\ x=0 \Rightarrow u=-1 \\ x=1 \Rightarrow u=2 \end{array}\right]$

$$\int_0^1 e^{3x-1}dx = \int_{-1}^2 e^u\cdot\frac{1}{3}du = \frac{1}{3}\int_{-1}^2 e^u\,du = \boxed{\frac{1}{3}\left[e^2-e^{-1}\right]}$$

(4) $\left[\begin{array}{l} u=2x^2-1 \\ du=4x\,dx \\ x=0 \Rightarrow u=-1 \\ x=1 \Rightarrow u=1 \end{array}\right]$

$$\int_0^1 xe^{2x^2-1}dx = \int_{-1}^1 e^u\cdot\frac{1}{4}du = \frac{1}{4}\int_{-1}^1 e^u\,du = \boxed{\frac{1}{4}\left[e^1-e^{-1}\right]}$$

(5) $\left[\begin{array}{l} u=\ln x \\ du=\frac{1}{x}dx \\ x=e \Rightarrow u=1 \\ x=e^2 \Rightarrow u=2 \end{array}\right]$

$F(u)=\ln u$

$$\int_e^{e^2}\frac{1}{\ln x}\cdot\frac{1}{x}dx = \int_1^2\frac{1}{u}\cdot du = \ln 2-\overset{0}{\cancel{\ln 1}} = \boxed{\ln 2}$$