HW 1 Solution

1. (a) Since (x_0, y_0) & (x_1, y_1) are on the line, we have two equations

$$\begin{cases} ① \; y_0 = m x_0 + b \\ ② \; y_1 = m x_1 + b \end{cases} \underset{①-②}{\Rightarrow} \quad y_0 - y_1 = m(x_0 - x_1) \Rightarrow m = \frac{y_0 - y_1}{x_0 - x_1}$$

Plug this into ① or ② & solve for b. Then $b = y_0 - \left(\frac{y_0 - y_1}{x_0 - x_1}\right) \cdot x_0$

So $y = mx + b$ is $y = \left(\frac{y_0 - y_1}{x_0 - x_1}\right) x + y_0 - \left(\frac{y_0 - y_1}{x_0 - x_1}\right) \cdot x_0$

$$\Rightarrow y - y_0 = \left(\frac{y_0 - y_1}{x_0 - x_1}\right)(x - x_0) = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_0). \Rightarrow \boxed{y - y_0 = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_0)}.$$

(b) (i) slope $= \frac{\text{Rise}}{\text{Run}} = \frac{y_1 - y_0}{x_1 - x_0}$.

(ii) Recall that the slope-point form is $y - y_0 = m(x - x_0)$. with P.

So $y - y_0 = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_0)$, the same as we got in (a).

(c) The slope-point form with Q is $y - y_1 = m(x - x_1)$ so we get

$\boxed{y - y_1 = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_1).}$ Let's check this equals (a).

$$y = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_1) + y_1 = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)x - x_1\left(\frac{y_1 - y_0}{x_1 - x_0}\right) + y_1 = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)x + \frac{(-x_1 y_1 + x_1 y_0) + y_1(x_1 - x_0)}{(x_1 - x_0)}$$

$$= \left(\frac{y_1 - y_0}{x_1 - x_0}\right)x + \frac{x_1 y_0 - y_1 x_0}{(x_1 - x_0)} \quad \text{———} \; (*)$$

From (a), $y = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_0) + y_0 = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)x - x_0\left(\frac{y_1 - y_0}{x_1 - x_0}\right) + y_0$

$$= \left(\frac{y_1 - y_0}{x_1 - x_0}\right)x + \frac{(-x_0 y_1 + x_0 y_0) + y_0(x_1 - x_0)}{(x_1 - x_0)} = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)x + \frac{y_0 x_1 - x_0 y_1}{(x_1 - x_0)} \quad \text{—} \; (**)$$

Then $(*)$ & $(**)$ are the same. So we get the same equations, which means we can take either P or Q.

2. (1) $y = mx \Rightarrow \boxed{y=-2x}$

(2) $y = mx+b \Rightarrow \boxed{y = 3x-5}$

(3) $y - y_0 = m(x-x_0) \Rightarrow y-(-4) = -2(x-(-1)) \Rightarrow y = -2x-2-4 \Rightarrow \boxed{y=-2x-6}$

(4) $y - y_0 = \left(\frac{y_1-y_0}{x_1-x_0}\right)(x-x_0) \Rightarrow y-2 = \frac{-4-2}{2-(-1)}(x-(-1)) \Rightarrow \boxed{y=-2x}$

(5) $x = a \Rightarrow \boxed{x=3}$

(6) $y = b \Rightarrow \boxed{y=-1}$

(7) $\boxed{x=0}$

(8) $\boxed{y=0}$

3. $l =$ (circumference of the circle of radius r) $\cdot$ (portion of θ radian out of $360° = 2\pi$)

$= (2\pi r)\cdot\left(\frac{\theta}{2\pi}\right) = r\theta.$

So $\boxed{l = r\theta}$.

4. (1) $\lim\limits_{x\to 2} \frac{x^2+7x+10}{x+2} = \frac{2^2+7\cdot 2+10}{2+2} = \frac{28}{4} = \boxed{7}$

(cf. $\lim\limits_{x\to -2} \frac{x^2+7x+10}{x+2} = \lim\limits_{x\to -2} \frac{\cancel{(x+2)}(x+5)}{\cancel{(x+2)}} = \lim\limits_{x\to -2}(x+5) = -2+5 = \boxed{3}$)

$\Rightarrow \left(\frac{0}{0}\right)$

(2) $\lim\limits_{x\to\infty} \frac{1}{x-1} = \frac{1}{\infty-1} = \frac{1}{\infty} = \boxed{0}$

(3) $\lim\limits_{x\to 1} \frac{1}{x-1}$ = undefined, since it is $\frac{1}{0}$ so either ∞ or $-\infty$.

(4) $\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x}{x} = \frac{\cos\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{0}{\frac{\pi}{2}} = \boxed{0}.$

(5) $\lim\limits_{x\to 0} \frac{\sin(3x)}{x} = \lim\limits_{x\to 0} \frac{\sin(3x)}{(3x)}\cdot 3 = \left(\lim\limits_{3x\to 0} \frac{\sin(3x)}{(3x)}\cdot\right) 3$

$= 3\cdot \lim\limits_{y\to 0}\frac{\sin y}{y}$ (by letting $y=3x$)

$= 3\cdot(1)$ (by theorem given in class)

$= \boxed{3}.$