Name : Solution

Student ID # :

Math 1080
Spring 2006
Instructor: Bo-Hae Im

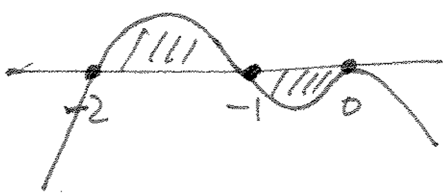
EXAM 3
Thursday, April 11, 2006

Problem	points	score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

1. Find the area of the region surrounded by the graph of $f(x) = -x^4 - 3x^3 - 2x^2$ and the x-axis.

$$f(x) = -x^2(x^2+3x+2) = -x^2(x+2)(x+1)$$

$$x = 0, -1, -2.$$



$$\text{Area} = \int_{-2}^{-1}(-x^4-3x^3-2x^2)\,dx - \int_{-1}^{0} -x^4-3x^3-2x^2\,dx$$

$$\left(F(x) = -\tfrac{1}{5}x^5 - \tfrac{3}{4}x^4 - \tfrac{2}{3}x^3.\right)$$

$$= [F(-1) - F(-2)] - [F(0) - F(-1)]$$

$$= 2F(-1) - F(-2) - \cancel{F(0)}$$

$$= 2\left[\frac{1}{5} - \frac{3}{4} + \frac{2}{3}\right] - \left[\frac{32}{5} - \frac{3\cdot \cancel{16}^{4}}{\cancel{4}} + \frac{2\cdot 8}{3}\right]$$

$$= 2\left[\frac{12-45+40}{60}\right] - \left[\frac{96+80}{15} - 12\right]$$

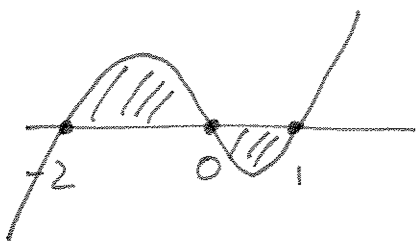
$$= \frac{7}{30} + 12 - \frac{176}{15}$$

$$= \frac{7+360-352}{30} = \frac{15}{30} = \frac{1}{2}.$$

2. Find the area of the region surrounded by the graph of $f(x) = x^3 + x^2 - 2x$ and the x-axis.

$$f(x) = x(x^2+x-2) = x(x+2)(x-1)$$

x-intercepts $= 0,\ 1,\ -2$



$$\text{Area} = \int_{-2}^{0} (x^3+x^2-2x)\,dx - \int_0^1 (x^3+x^2-2x)\,dx$$

$$\left(F(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2\right)$$

$$= \left(\cancel{F(0)} - F(-2)\right) - \left(F(1) - \cancel{F(0)}\right)$$

$$= -F(-2) - F(1)$$

$$= -\left[\frac{16}{4} - \frac{8}{3} - 4\right] - \left[\frac{1}{4} + \frac{1}{3} - 1\right]$$

$$= -\frac{17}{4} + \frac{7}{3} + 5$$

$$= \frac{60 - 51 + 28}{12}$$

$$= \frac{37}{12}$$

3. Find $F(x)$ such that $F'(x) = 3x^2 - 4x + 3$ and $F(1) = 5$.

$$F(x) = x^3 - 2x^2 + 3x + C$$

$$5 = F(1) = 1 - 2 + 3 + C$$

$$\Rightarrow \quad C = 3$$

So $\boxed{F(x) = x^3 - 2x^2 + 3x + 3}$

4. Find $\int_1^{e^3} -\frac{2}{x}\,dx$ and simplify your answer as much as you can.

$F(x) = -2\ln x.$

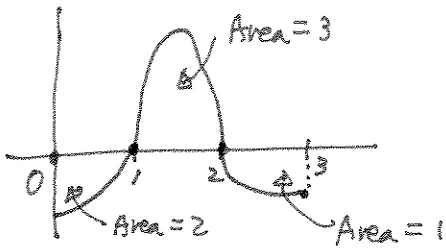
So $\displaystyle\int_1^{e^3} -\frac{2}{x}\,dx = F(e^3) - F(1)$

$= -2\ln e^3 - (\cancel{-2\cdot\ln 1})$

$= -2\cdot 3\cdot \ln e$

$= -6$

5. The graph of the derivative $f'(x)$ of $f(x)$ on $[0, 4]$ is given below. Each area between f' and the x-axis on each interval is given below. Suppose $f(1) = 3$. Find $f(3)$ and $f(0)$.



⟨graph of $y=f'(x)$⟩

(1) $$f(3) - f(1) = \int_1^3 f'(x)\,dx$$

$$f(3) - 3 = 3 - 1 = 2 \Rightarrow \boxed{f(3) = 5}$$

(2) $$f(1) - f(0) = \int_0^1 f'(x)\,dx$$

$$3 - f(0) = -2 \Rightarrow f(0) = 5$$

6. Let $f(x) = -x + 2$ on $[1, 3]$.

(1) Find $\mathbf{RS}_n$.

(2) Find $\int_1^3 f(x)dx$ by using (1).

(1) $\Delta x = \frac{2}{n}$, $\quad x_k = 1 + k \cdot \frac{2}{n}$.

$$RS_n = \sum_{k=1}^{n} f(x_k) \cdot \Delta x = \frac{2}{n} \cdot \sum_{k=1}^{n} \left(-1 - k \cdot \frac{2}{n} + 2\right)$$

$$= \frac{2}{n} \sum_{k=1}^{n} \left(1 - \frac{2}{n} \cdot k\right)$$

$$= \frac{2}{n} \left(n - \frac{\cancel{2}}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}} \right)$$

$$= \frac{2}{n} \left(\cancel{n} - \cancel{n} - 1 \right) = -\frac{2}{n}$$

(the definition of $\int$)

(2) $$\int_1^3 f(x)\,dx = \lim_{n \to \infty} RS_n$$

$$= \lim_{n \to \infty} -\frac{2}{n} = \boxed{0}.$$