Name : Solution

Student ID # :__________

Math 1080
Spring 2006
Instructor: Bo-Hae Im

EXAM 2
Thursday, March 9, 2006

Problem	points	score
1	16	
2	10	
3	8	
4	10	
5	8	
6	8	
EC	5	
Total	60(+5)	

1. Let f have its derivative $f'(x) = -(x+1)(x-1)^2$.

(1) (3 pts) Find the x-coordinates of critical points of f if any.

$f'(x) = 0. \Rightarrow$ $x = -1, \ 1$

(or $(-1, f(-1))$ & $(1, f(1))$ are critical points)

(2) (6 pts)Find the x-coordinates of inflection points of f if any. Explain why they are inflection points.

$f''(x) = (f'(x))' = -(x-1)^2 - (x+1)\cdot 2(x-1)\cdot 1 = (x-1)(-x+1-2x-2)$

$= (x-1)(-3x-1) = 0 \Rightarrow x = 1, -\frac{1}{3}$

		$-\frac{1}{3}$		1	
f''	$-$		$+$		$-$
	CD		CU		CD

So f changes concavity at both 1 & $-\frac{1}{3}$.

So $\boxed{x = 1, -\frac{1}{3}}$

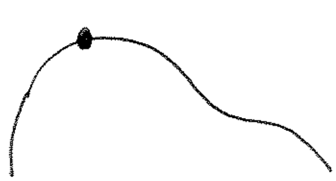
(or $(1, f(1))$ and $(-\frac{1}{3}, f(-\frac{1}{3}))$ are inflection points)

(3) (7 pts) Find the x-coordinates of local maxima and local minima of f respectively, if any.

		-1		$-\frac{1}{3}$		1	
f'	$+$	0	$-$	$-$	$-$	0	$-$
f''	$-$	$-$	$-$	0	$+$	0	$-$
f							

f has a local max at $x = -1$ but no local min.

f



2.(10 pts) Show that the equation $2x^3 + 2x - 1 = 0$ has exactly one and only one real solution by mentioning the names of theorems that you apply.

First, let $f(x) = 2x^3 + 2x - 1$.

$f(0) = -1 < 0$
$f(1) = 2+2-1 > 0$.

So by Intermediate Value Theorem, since $f(0) < 0 < f(1)$ and f is continuous, there exists c between 0 and 1 such that $f(c) = 0$.

So c is a solution of $2x^3 + 2x - 1 = 0$.

Now, we need to show there exists only one solution.

Suppose not. That is, there exist two solutions c_1 and c_2. Then $f(c_1) = 0 = f(c_2)$.

Then by Rolle's theorem (or Mean Value Theorem), there exists d between c_1 and c_2 such that $f'(d) = 0$.

But $f'(x) = 6x^2 + 2 \neq 0$ so $f'(d) \neq 0$.

We got the contradiction.

So we conclude that f cannot have more than one solution. So f has only one solution.

3.(8 pts) Find an approximation of the solution to the equation $2x^3 - x - 5 = 0$ with accuracy 0.3 by using Bisection method. You might need the following values of approximations.

$0.25^3 \approx 0.0156$
$0.5^3 = 0.125$
$0.75^3 \approx 0.422$
$1.25^3 \approx 1.953$
$1.5^3 = 3.375$
$1.75^3 \approx 5.359$
$2^3 = 8$
$2.25^3 \approx 11.39$
$2.5^3 = 15.625$
$2.75^3 \approx 20.8$
$3^3 = 27$

Let $f(x) = 2x^3 - x - 5$.

$f(0) = -5 < 0$
$f(1) = 2-1-5 < 0$
$f(2) = 2\cdot 8 - 2 - 5 = 16-2-5 > 0.$

$\Rightarrow$ there exists a solution between 1 and 2 by IVT.
Let $a_1 = 1$ & $b_1 = 2$.

n	a_n	b_n	$f(a_n)$	$f(b_n)$	m_n	$f(m_n)$	h_n	
1	1	2	$-$	$+$	1.5	$+$	0.5	> 0.3
2	1	1.5	$-$	$+$	$\boxed{1.25}$	$-$	0.25	< 0.3

So $\boxed{1.25}$ is an approximation of a solution of $2x^3 - x - 5 = 0$ between 1 and 2.

4.(10 pts) Find two numbers whose product is -9 and the sum of whose squares is a minimum.

Let x & y be two numbers whose product is -9.

Then $xy = -9 \Rightarrow y = \frac{-9}{x}$.

Let $f(x) = x^2 + y^2 = x^2 + \left(\frac{-9}{x}\right)^2 = x^2 + \frac{81}{x^2} = x^2 + 81 \cdot x^{-2}$.

$$\Rightarrow f'(x) = 2x - 2 \cdot 81 \cdot x^{-3} = 2x - \frac{162}{x^3}$$

$$= \frac{2x^4 - 162}{x^3} \Rightarrow x = 0, \quad 2x^4 - 162 = 0.$$

$$2(x^4 - 81) = 2(x^2-9)(x^2+9) = 2(x+3)(x-3)(x^2+9)$$

Critical points are at $x = 0, 3, -3$.

		-3		0		3	
f'	$-$		$+$		$-$		$+$
	$\searrow$		$\nearrow$		$\searrow$		$\nearrow$

The global min of f must be the smaller one of local minimums.

So $f(-3) = (-3)^2 + \frac{81}{(-3)^2} = 9 + \frac{81}{9} = 9 + 9 = 18$

$f(3) = (+3)^2 + \frac{81}{3^2} = 18$ Same.

So $\begin{cases} x = 3 \\ y = -3 \end{cases}$ or $\begin{cases} x = -3 \\ y = 3 \end{cases}$

So two numbers $\boxed{3 \ \& \ -3}$ are such a pair.

5.(8 pts) When is the function $f(x) = -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 1$ increasing? Find the range of x at f is increasing.

We need to find when $f'(x) > 0$.

$$f'(x) = -x^2 + x + 2$$
$$= -(x^2 - x - 2)$$
$$= -(x-2)(x+1) > 0$$

$$\Rightarrow \quad (x-2)(x+1) < 0$$
$$\boxed{-1 < x < 2}$$

So $\boxed{\text{f is increasing if } -1 < x < 2.}$

(or you can use the table also.)

6.(8 pts) Evaluate the following:

$\sum_{k=1}^{30}(k^2 - k - 2) =$ $\sum_{k=1}^{30} k^2 - \sum_{k=1}^{30} k - \sum_{k=1}^{30} 2$

$= \frac{30\cdot(30+1)\cdot(30\cdot 2+1)}{6} - \frac{30\cdot(30+1)}{2} - 2\cdot 30$

$= \frac{\cancel{30}^{5}\cdot 31\cdot 61}{\cancel{6}} - \frac{\cancel{30}^{15}\cdot 31}{\cancel{2}} - 60$

$= \boxed{8930}$

(Extra-credit problem, 5 pts, NO partial credit. The full credit will be given only if you show all of your work and your work is a mathematically complete one.)

If $f'(x) = 3g'(x)$ for all x between 1 and 5, and $f(2) = 1, g(2) = 2$, and $g(3) = 3$, then find $f(3)$.

Let $h(x) = f(x) - 3g(x)$.

Then $h'(x) = f'(x) - 3g'(x) = 0$.

So $h(x) = C$ on $[1, 5]$.

Then $h(2) = f(2) - 3g(2)$

$= 1 - 3 \cdot 2 = -5$

So $h(x) = -5$ for all x on $[1, 5]$.

So $h(3) = -5$.

Then $f(3) = h(3) + 3 \cdot g(3)$

$= -5 + 3 \cdot 3$

$= -5 + 9 = \boxed{4}$