Name : Solution

Student ID # :

Math 1080
Spring 2006
Instructor: Bo-Hae Im

EXAM 1
Thursday, February 9, 2006

Problem	points	score
1	8	
2	12	
3	6	
4	8	
5	18	
6	8	
EC	5	
Total	60(+5)	

1. (8 pts) Find $f'(x)$ of $f(x) = -x^3 + 1$ by using the limit definition (no credit will be given if you use the derivative rule/formula immediately).

$$f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}$$

$$= \lim_{t \to x} \frac{(-t^3+1) - (-x^3+1)}{(t-x)}$$

$$= \lim_{t \to x} \frac{-t^3 + x^3}{(t-x)}$$

$$= \lim_{t \to x} -\frac{(t^3 - x^3)}{(t-x)}$$

$$= \lim_{t \to x} -\frac{\cancel{(t-x)}(t^2+tx+x^2)}{\cancel{(t-x)}}$$

$$= \lim_{t \to x} -(t^2+tx+x^2)$$

$$= -(x^2 + x \cdot x + x^2)$$

$$= \boxed{-3x^2}.$$

(And you can check that $(-x^3+1)' \stackrel{?}{=} -3x^2$ by the rule!)

2. (each 4 pts) Evaluate the following (show your work).

(1) $\lim_{x\to 0} \frac{\sin(5x)}{x} = \lim_{x\to 0} \frac{\sin(5x)}{(5x)} \cdot \frac{5}{1} = 5 \cdot \lim_{x\to 0} \frac{\sin(5x)}{(5x)}$

$= 5 \lim_{y\to 0} \frac{\sin(y)}{(y)}$ (since for $y=5x$, $y\to 0$ as $x\to 0$)

$= 5 \cdot 1 = \boxed{5}$

(2) $\lim_{x\to 2} \frac{x^2+3x-10}{x-2} = \lim_{x\to 2} \frac{(x+5)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x\to 2} (x+5) = 2+5 = 7$

(3) $\log_2 8 - 2e^{\ln(\frac{1}{2})} = \log_2 2^3 - 2\, \cancel{e^{\ln}} \cdot \frac{1}{2}$

$= 3 \log_2 2 - 2 \cdot \frac{1}{2}$

$= 3 \cdot 1 - 1$

$= 3 - 1 = 2$

3. (6 pts) Approximate $(0.99)^8$ by using the linear approximation method. (Your approximation should be less than 1 obviously.)

let $f(x) = x^8$. $f'(x) = 8x^7$.

& $1 \sim 0.99$

$$(0.99)^8 = f(0.99) \approx f(1) + f'(1)(0.99-1)$$

$$= 1^8 + 8 \cdot 1^7 (-0.01)$$

$$= 1 + 8 \cdot (-0.01)$$

$$= 1 - 0.08$$

$$= \boxed{0.92}$$

4. (8 pts) Find the tangent line of the curve $f(x) = \sin(x) - e^{2x}$ at $x = 0$.

Point $= (0, f(0)) = (0, \sin(0) - e^{0})$

$= (0, -1)$

$f'(x) = \cos(x) - 2e^{2x}$.

$\Rightarrow$ slope $= f'(0) = \cos(0) - 2 \cdot e^{0} = 1 - 2 = -1$

Equation is $y - y_0 = m(X - x_0)$.

$\Rightarrow$ $y - (-1) = -1(x - 0)$

$\Rightarrow$ $\boxed{y + 1 = -x.}$

or $(y = -x - 1)$

5. (each 6 pts) Find the derivatives $f'(x)$ of the following $f(x)$ by using the derivative rules.

(1) $f(x) = \frac{x^2 - x - 2}{x+1}$ $= \frac{(x-2)(x+1)}{(x+1)} = x-2. \Rightarrow f'(x) = (x-2)' = 1-0 = 1$

So $\boxed{f'(x) = 1}$ ^^.

$\left(\text{or } f'(x) = \frac{(2x-1)(x+1) - (x^2-x-2)\cdot 1}{(x+1)^2} = \frac{2x^2+2x-x-1-x^2+x+2}{(x+1)^2} \right.$

$\left. = \frac{x^2+2x+1}{(x+1)^2} = \frac{(x+1)^2}{(x+1)^2} = 1 \quad \text{the same!} \right)$

(2) $f(x) = (\cos(2x) - 1)^3$

$f'(x) = 3\cdot(\cos(2x)-1)^{3-1}\cdot(\cos(2x)-1)'$

$= 3\,(\cos(2x)-1)^2\cdot(-\sin(2x)\cdot 2 - 0)$

$= \boxed{-6\sin(2x)\cdot(\cos(2x)-1)^2}$

(3) $f(x) = e^{x^2} \cdot \ln(x)$

$f'(x) = (e^{x^2})'\cdot\ln(x) + e^{x^2}\cdot(\ln(x))'$

$= \boxed{e^{x^2}\cdot(2x)(\ln x) + e^{x^2}\cdot\frac{1}{x}}$

$= 2x\,e^{x^2}\ln x + \frac{e^{x^2}}{x}.$

6. (8 pts) An object is moving along the horizontal line according to the position function $s(t) = -t^3 + 6t^2 - 9t - 1$ with respect to time t. When is the object moving forwards (to the right) faster and faster?

We need to solve $v(t) > 0$ and $a(t) > 0$,

Since the graph of $S(t)$ must be C.U. & Increasing.

$$\Rightarrow \quad v(t) = -3t^2 + 12t - 9 = -3(t^2 - 4t + 3) = -3(t-3)(t-1).$$

$$\& \quad a(t) = -6t + 12 = -6(t-2).$$

Solve $\quad -3(t-3)(t-1) > 0 \quad$ and $\quad -6(t-2) > 0.$

$$\Leftrightarrow \quad (t-3)(t-1) < 0 \quad \text{and} \quad (t-2) < 0.$$

$$\Leftrightarrow \quad 1 < t < 3 \quad \text{and} \quad t < 2.$$

$$\Leftrightarrow \quad \boxed{1 < t < 2}$$

(Extra-credit problem, 5 pts, NO partial credit. The full credit will be given only if you show all of your work and your work is a mathematically complete one.)

Find the following by using the only facts and rules taught in class.

$$\lim_{x\to 0}\frac{\sin(2x)}{\tan(5x)} = \lim_{x\to 0}\frac{\sin(2x)}{\frac{\sin(5x)}{\cos(5x)}} = \lim_{x\to 0}\sin(2x)\cdot\cos(5x)\cdot\frac{1}{\sin(5x)}$$

$$= \lim_{x\to 0}\frac{\sin(2x)}{(2x)}\cdot(2x)\cdot\cos(5x)\cdot\frac{(5x)}{\sin(5x)}\cdot\frac{1}{(5x)}$$

This explanation (*) is necessary to get the full credit!

$$= \boxed{\lim_{x\to 0}\frac{\sin(2x)}{(2x)}}\cdot\lim_{x\to 0}\frac{2x}{5x}\cdot\lim_{x\to 0}\cos(5x)\cdot\boxed{\lim_{x\to 0}\frac{1}{\frac{\sin(5x)}{(5x)}}}$$

$\begin{pmatrix} y=2x \\ t=5x \end{pmatrix}$

$$= \lim_{y\to 0}\frac{\sin(y)}{(y)}\cdot\lim_{x\to 0}\frac{2}{5}\cdot\lim_{x\to 0}\cos(5x)\cdot\frac{1}{\lim_{t\to 0}\frac{\sin(t)}{(t)}}$$

$$= 1\cdot\frac{2}{5}\cdot\cos(0)\cdot\frac{1}{1}$$

$$= 1\cdot\frac{2}{5}\cdot 1\cdot 1$$

$$= \boxed{\frac{2}{5}}$$