

Review Problems

Midterm 2

December 8, 2007

1. **Problem 1** - Find the indicated derivative of the following functions.

(a) Find $\frac{dy}{dx}$ if $y = \frac{x}{\sqrt[3]{3x-1}}$.

Answer: $y' = \frac{2x-1}{(3x-1)^{4/3}}$

(b) If $\frac{dy}{dx} = \sqrt{x^2-4}$, find $\frac{d^3y}{dx^3}$.

Answer: $y''' = \frac{-4}{(x^2-4)^{3/2}}$

(c) Find $\frac{dy}{dt}$ if $y = \ln\left(\frac{t^2}{\sqrt{5-2t}}\right)$.

Answer: $y' = \frac{2}{t} - \frac{1}{(5-2t)}$

(d) Find $\frac{dy}{dt}$ if $y = \left[\ln\left(\frac{3t}{t-1}\right)\right]^2$.

Answer: $y' = \frac{-2}{t(t-1)} \ln\left(\frac{3t}{t-1}\right)$

2. **Problem 2** - The function and its first and second derivatives are given. Use these to find any horizontal and vertical asymptotes, critical points, relative maxima, relative minima, and point of inflection. Then sketch the graph of the function.

$$\begin{aligned}
 y &= \frac{x^2}{(x-3)^2} \\
 y' &= \frac{-6x}{(x-3)^3} \\
 y'' &= \frac{12x+18}{(x-3)^4}
 \end{aligned}$$

Answer: VA: $x = 3$; HA: $y = 1$; critical point: $(0, 0)$; relative min: $(0, 0)$; POI: $(-3/2, 1/9)$

3. **Problem 3** - Profit

Suppose that in a monopolistic market, the demand function for a commodity is

$$p = 7000 - 10x - \frac{x^2}{3}$$

where x is the number of units and p is the price (in dollars). If a company's average cost function for this commodity is

$$\overline{C}(x) = \frac{40,000}{x} + 600 + 8x$$

find the maximum profit.

Answer: Profit $P(x) = 6400x - 18x^2 - \frac{x^3}{3} - 40,000$; $x = 64$ maximizes the profit; $P(64) = 208,490.67$;

4. **Problem 4** - Problem 21 page 740

Answer: Dimension of the box: length = 8, width = 4, height = 8.

5. **Problem 5** - Problem 9 page 729

Answer: 40 people will maximize the revenue.

6. **Problem 6** - Use second derivative test to find the relative maxima, relative minima, and point of inflection, and sketch the graph of

$$y = x^4 - 8x^3 + 16x^2$$

Answer: relative min: $(0, 0)$ and $(4, 0)$; relative max: $(2, 16)$; POI: $(2 - \frac{2}{\sqrt{3}}, 7.111)$ and $(2 + \frac{2}{\sqrt{3}}, 7.111)$