Review Problems

Midterm 1

September 18, 2007

1. **Problem 1** - Find each limit, if it exists.
(a)

$$\lim_{x \to -2} f(x) \text{ where } f(x) = \begin{cases} x^3 - x & \text{if } x \leq -2 \\ 2 - x^2 & \text{if } x > -2 \end{cases}$$

Answer: DNE
(b)

$$\lim_{x \to 4} f(x) \text{ where } f(x) = \frac{x^2 - 16}{x - 4}$$

Answer: 8
(c) What does the answer in part (b) tell you about the vertical asymptote for the graph of $f(x)$.

Answer: No vertical asymptote

2. **Problem 2**
(a) Determine if the function is continuous. If it is not, identify where it is discontinuous and which condition fails to hold.

$$f(x) = \begin{cases} x+2 & \text{if } x \leq -2 \\ 5x-6 & \text{if } x > -2 \end{cases}$$

Answer: Discontinuous at $x = -2$
(b) Evaluate the limit if it exists. What does the answer tell you about the horizontal asymptote of the graph of the function?

$$\lim_{x \to \infty} \frac{7x^2 - 3x}{5x + 4}$$

Answer:DNE. No horizontal asymptote

3. **Problem 3** - Use the definition of derivative to find $f'(x)$ if $f(x) = 4x^2 - 2x + 1$.

Answer:$8x - 2$

4. **Problem 4**-Marginal Revenue
Suppose the total revenue function for a pizza is

$$R(x) = 20x - 0.2x^2 \text{ dollars}$$

where x is the number of pizzas sold.
(a) What is the function that gives marginal revenue?

Answer:$20 - 0.4x$

(b) What is the marginal revenue when 50 pizzas are sold, and what does it mean?

Answer:0

5. **Problem 5**- Find the derivative of each function.
   (a)
   $$g(x) = \frac{7}{x^7} - \frac{3}{x^3} + 8\sqrt{x}$$

   Answer:$-49x^{-8} + 9x^{-4} + 4x^{-1/2}$
   (b)
   $$f(x) = (x^5 + 1)(x^2 - 3)$$

   Answer:$7x^6 - 15x^4 + 2x$

6. **Problem 6**-Given
   $$f(x) = \frac{x^2 - 2x}{x + 3}$$

   (a) Find $\frac{df}{dx}$.

   Answer:$\frac{x^2+6x-6}{(x+3)^2}$
   (b) Find the slope of the tangent to the graph of $f(x)$ at $x = -1$.

Answer:$-11/4$

(c) Write the equation of the tangent line to the graph of $f(x)$ at $(-1, 3/2)$.

Answer:$y = -\frac{11}{4}x - 5/4$

7. **Problem 7**- Given
$$f(x) = \frac{2\sqrt{x^3} - 1}{3 - \sqrt[3]{x}}$$

(a) Find $f'(x)$.

Answer:
(b) What is the instantaneous rate of change of $f(x)$ at $x = 1$?

Answer:19/12

8. **Problem 8** - Problem 13 (a-h) page 683 in your book.

Answer: (a) -5; (b) -1; (c) 4; (d) DNE; (e) 2; (f) 3/2
(g) -4, 1, 3, 6
(h) -4, 3, 6.