MATH 4200-1 FALL 2008

Fourth Mock Exam

INSTRUCTOR: H.-PING HUANG

LAST NAME _____

FIRST NAME _____

ID NO. _____

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1	25	_____
PROBLEM 2	25	_____
PROBLEM 3	25	_____
PROBLEM 4	25	_____
PROBLEM 5	25	_____
PROBLEM 6	25	_____
PROBLEM 7	25	_____
PROBLEM 8	25	_____
TOTAL	200	_____

PROBLEM 1

(25 pt) Analyze the function $\sqrt{z}$ defined by

$$\sqrt{z} = e^{(1/2)\log z}$$

using the principal branch of the log function. What kind of a jump, if any, does it have as z crosses the negative real axis?

PROBLEM 2

(25 pt) Prove that if f is an entire function which satisfies $|f(z)| \geq 1$ on the entire plane, then f is constant.

Prove that if an entire function has real part which is bounded above, then the function is constant.

PROBLEM 3

(25 pt) Suppose f is analytic in the annulus $A = \{z \in \mathbb{C} : R < |z|\}$ and satisfies the inequality $|f(z)| \leq |z|^k$ in the set. Then prove the Laurent expansion of f in A has no terms with positive exponent greater than k.

PROBLEM 4

(25 pt) Derive a formula for $\int_\gamma \tan z dz$, where γ is any closed path which does not path through an integral multiple of π.

Use long division of power series to find the power series expansion about 0 of $\tan z = \sin z / \cos z$ through terms of degree five.

PROBLEM 5

(25 pt) Find

$$\int_{-\infty}^{\infty} \frac{x}{(x^2+2x+2)^2} dx$$

PROBLEM 6

(25 pt) Find a conformal equivalence from the set

$$A = \{z : -\pi/2 < \text{Re } (z) < \pi/2, \quad \text{Im } (z) > 0\}$$

to the upper half plane.

PROBLEM 7

(25 pt) Find the image of the unit disc under the transformation

$$h(z) = \frac{2z}{z - i}.$$

PROBLEM 8

(25 pt) Find a conformal automorphism of D which takes $1/2$ to 0 and has derivative $3i/4$ at $z = 0$.