MATH 3220-1 FALL 2008

Fourth Mock Exam

INSTRUCTOR: H.-PING HUANG

LAST NAME ______________________________

FIRST NAME ______________________________

ID NO. ______________________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1	25	______
PROBLEM 2	25	______
PROBLEM 3	25	______
PROBLEM 4	25	______
PROBLEM 5	25	______
PROBLEM 6	25	______
PROBLEM 7	25	______
PROBLEM 8	25	______
TOTAL	200	______

PROBLEM 1

(25 pt) Prove that every convergent sequence in $\mathbb{R}^d$ is bounded.

PROBLEM 2

(25 pt) Des the series $\sum_{k=0}^{\infty} x^k y^k$ converge uniformly on the square

$$\{(x,y) \in \mathbb{R}^2 : -1 < x < 1, -1 < y < 1\}?$$

Justify your answer.

Des the series $\sum_{k=0}^{\infty} x^k y^k$ converge uniformly on the disc

$$\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}?$$

Justify your answer.

PROBLEM 3

(25 pt) For the function

$$f(x,y) = \begin{cases} \frac{x^3y - xy^3}{x^2+y^2} & (x,y) \neq (0,0), \\ 0 & (x,y) = (0,0) \end{cases}$$

show that the first order partial derivatives exist and are continuous everywhere. Is it differentiable?

PROBLEM 4

(25 pt) Find the degree $n = 2$ Taylor's formula for $f(x, y) = \ln(x + y)$ at the point $a = (0, 1)$.

PROBLEM 5

(25 pt) Find all point $(r, \theta) \in \mathbb{R}^2$ such that the polar change of coordinates function

$$F(r, \theta) = (r \cos \theta, r \sin \theta)$$

has a smooth local inverse at a. Find the inverse and its differential at one such point.

PROBLEM 6

(25 pt) Suppose that f and g are functions defined on an aligned rectangle R. Suppose there is a positive constant K such that $|f(x) - f(y)| \leq K|g(x) - g(y)|$ for all $x, y \in R$. Prove that if g is integrable on R, then so is f.

PROBLEM 7

(25 pt) Let A be a Jordan region and f an integrable function on A. The average value f on A is defined to be the number

$$\text{avg }(f, A) = \frac{1}{V(A)} \int_A f(x) dV(x).$$

If A is compact and connected and f is continuous on A, prove that there is a point $x_0 \in A$ at which $f(x_0) = \text{avg }(f, A)$.

PROBLEM 8

(25 pt) Prove that if $f(t,x)$ is a continuous function on $I \times A$, where I is an open region and A is a compact Jordan region in $\mathbb{R}^d$, and if $f_t(t,x)$ exists and is continuous on $I \times A$, then

$$\frac{d}{dt}\int_A f(t,x)dV(x) = \int_A f_t(t,x)dV(x).$$

If A is not compact, what would happen? Can you produce a counterexample?