MATH 3220-1 FALL 2008
Fourth Mock Exam
INSTRUCTOR: H.-PING HUANG

LAST NAME ____________________________
FIRST NAME __________________________
ID NO. ________________________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE SPECIFIED METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1  25 ______
PROBLEM 2  25 ______
PROBLEM 3  25 ______
PROBLEM 4  25 ______
PROBLEM 5  25 ______
PROBLEM 6  25 ______
PROBLEM 7  25 ______
PROBLEM 8  25 ______
TOTAL  200 ______
PROBLEM 1

(25 pt) Prove that every convergent sequence in $\mathbb{R}^d$ is bounded.
PROBLEM 2

(25 pt) Des the series $\sum_{k=0}^{\infty} x^k y^k$ converge uniformly on the square
$$\{(x, y) \in \mathbb{R}^2 : -1 < x < 1, -1 < y < 1\}?$$

Justify your answer.

Des the series $\sum_{k=0}^{\infty} x^k y^k$ converge uniformly on the disc
$$\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}?$$

Justify your answer.
PROBLEM 3

(25 pt) For the function

\[ f(x, y) = \begin{cases} \frac{x^3y - xy^3}{x^2 + y^2} & (x, y) \neq (0, 0), \\ 0 & (x, y) = (0, 0) \end{cases} \]

show that the first order partial derivatives exist and are continuous everywhere. Is it differentiable?
PROBLEM 4

(25 pt) Find the degree $n = 2$ Taylor’s formula for $f(x, y) = \ln(x + y)$ at the point $a = (0, 1)$. 
PROBLEM 5

(25 pt) Find all points \((r, \theta) \in \mathbb{R}^2\) such that the polar change of coordinates function
\[
F(r, \theta) = (r \cos \theta, r \sin \theta)
\]
has a smooth local inverse at \(a\). Find the inverse and its differential at one such point.
PROBLEM 6

(25 pt) Suppose that $f$ and $g$ are functions defined on an aligned rectangle $R$. Suppose there is a positive constant $K$ such that $|f(x) - f(y)| \leq K|g(x) - g(y)|$ for all $x, y \in R$. Prove that if $g$ is integrable on $R$, then so is $f$. 

PROBLEM 7

(25 pt) Let $A$ be a Jordan region and $f$ an integrable function on $A$. The average value $f$ on $A$ is defined to be the number

$$\text{avg} \ (f, A) = \frac{1}{V(A)} \int_A f(x)dV(x).$$

If $A$ is compact and connected and $f$ is continuous on $A$, prove that there is a point $x_0 \in A$ at which $f(x_0) = \text{avg} \ (f, A)$. 
PROBLEM 8

(25 pt) Prove that if \( f(t, x) \) is a continuous function on \( I \times A \), where \( I \) is an open region and \( A \) is a compact Jordan region in \( \mathbb{R}^d \), and if \( f_t(t, x) \) exists and is continuous on \( I \times A \), then

\[
\frac{d}{dt} \int_A f(t, x) dV(x) = \int_A f_t(t, x) dV(x).
\]

If \( A \) is not compact, what would happen? Can you produce a counterexample?