MATH 3220-1 FALL 2008

Second Mock Exam

INSTRUCTOR: H.-PING HUANG

LAST NAME ______________________________

FIRST NAME ______________________________

ID NO. ______________________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1	25	______
PROBLEM 2	25	______
PROBLEM 3	25	______
PROBLEM 4	25	______
TOTAL	100	______

PROBLEM 1

(25 pt) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = \begin{cases} \frac{y^2 - x^2 y}{|y - x^2|} & \text{if } \; y \neq x^2 \\ 0 & \text{if } \; y = x^2 \end{cases}$$

At which points of $\mathbb{R}^2$ is this function continuous?

PROBLEM 2

(25 pt) Is the image of a closed set under a continuous function necessarily closed? Prove that it is or give an example where it is not.

Is the image of an open set under a continuous function necessarily open? Prove that it is or give an example where it is not.

PROBLEM 3

(25 pt) Prove that the following limit exists and evaluate it.

$$\lim_{n\to\infty}\int_0^{\pi/2}\sqrt{\sin\frac{x}{n}+\cos\frac{x}{n}}\;dx$$

PROBLEM 4

(25 pt) Des the series $\sum_{k=0}^{\infty} x^k y^k$ converge uniformly on the square

$$\{(x, y) \in \mathbb{R}^2 : -1 < x < 1, -1 < y < 1\}?$$

Justify your answer.

Des the series $\sum_{k=0}^{\infty} x^k y^k$ converge uniformly on the disc

$$\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}?$$

Justify your answer.

Bonus

(25 pt) Suppose that $T \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)$ for some $n, m \in \mathbb{N}$.
(a) If $T(1,1) = (3, \pi, 0)$ and $T(0,2) = (4,0,1)$, find the matrix representative of T.
(b) If $T(1,1,0) = (e, \pi)$, $T(0,-1,1) = (1,0)$, and $T(1,1,-1) = (4,7)$, find the matrix representative of T.