Solution Manual

#1 $\frac{\partial f}{\partial y} = x \cdot 6y + 6y^2 = 0$ (1)

when $x = -y$ or $y = 0$

$\downarrow$ $\downarrow$

$\left(\frac{1}{\sqrt[3]{2}}, -\frac{1}{\sqrt[3]{2}}\right)$ $(1, 0)$

#2 $\frac{\partial f}{\partial z} = x + y + \cos(x+y+z) \Big|_{(0,0,0)} = 1 \neq 0$

Ans: Yes

#3 $\begin{bmatrix} 2u & 2v \\ x & y \end{bmatrix}$

$\Delta \neq 0$ when $uy \neq vx$

#4 $\begin{bmatrix} 2u+2 & 2v \\ 3u^2 - x & \cos v + y \end{bmatrix}$

$\Delta = 2 \neq 0$

#5 $\begin{bmatrix} 3u^2 & 2x^2 v & 1 \\ 2y^2 u & 3v^2 & 1 \\ 0 & 0 & 2w + x \end{bmatrix}$

$\Delta = 5 \neq 0$

#6 $df = \begin{bmatrix} y+z \\ x+z \\ x+y \end{bmatrix}$ (2)

rank = 1 $\Rightarrow$ $f(x,y,z)=1$ is a smooth 2-surface

if only if not

$x = -y$

$y = -z$

$z = -x$

i.e. $(0,0,0)$

Hower $(0,0,0) \notin S$

tangent plane: $(b+c)(x-a) + (a+c)(y-b)$

$+ (a+b)(z-c) = 0$

i.e. $(b+c)x + (a+c)y + (a+b)z = 2$

#7 $dF = \begin{bmatrix} 2x & 2y & -2z \\ 1 & 1 & 1 \end{bmatrix}$

rank = 2 if $(x,y,z) \neq (0,0,0)$

tangent vector = $\begin{vmatrix} i & j & k \\ 2x & 2y & -2z \\ 1 & 1 & 1 \end{vmatrix}$

$= [2(y+z), -2(x+z), 2(x-y)]$

Another way:

$$x^2 + y^2 = z^2$$

$$x + y = -z \Rightarrow x^2 + y^2 + 2xy = z^2$$

$\Rightarrow 2xy = 0$ either $x = 0$ or $y = 0$

when $x = 0$ $y = -z$ & $y^2 - z^2 = 0$

$(0, -z, z) = S =$ line

when $y = 0$ $(-z, 0, z) = S =$ line

#8

$$\begin{bmatrix} 2x & 2y & 2u & -3 \\ 2+y & x-1 & 6u & -9 \end{bmatrix} = dF$$

If rank $= 2 \Rightarrow S$ is a smooth 2 surface

if only if not $2x = 3(2+y)$

$2y = 3(x-1)$

$(x, y) = (-\frac{3}{5}, \frac{12}{5})$

cannot solve u, v

#9

$$\begin{bmatrix} e^u & e^u & xe^u + ye^u & 0 \\ v & u & y & x \end{bmatrix} = dF$$

If rank $= 2 \Rightarrow S$ is a smooth 2-surface

If not, $u = v$, $x(x+y) = 0$, $y = v(x+y)$