SAS: $GL_2(\mathbb{F}_3) \circlearrowright \mathbb{F}_3^2 \times \mathbb{F}_3^2$

(i.e. pairs of column vectors $\vec{v}_1, \vec{v}_2$)

Q: How many orbits are there for this action?

A: 6.

First orbit: $(\vec{v}_1, \vec{v}_2)$ s.t. $\vec{v}_1$ & $\vec{v}_2$ are linearly independent.

If $(\vec{v}_1, \vec{v}_2)$ are lin. ind. & $(\vec{w}_1, \vec{w}_2)$ are lin ind then

$\exists!$ linear transformation sending $\vec{v}_1$ to $\vec{w}_1$ and $\vec{v}_2$ to $\vec{w}_2$ and its invertible.

Second orbit: $\{(\vec{0}, \vec{0})\}$

3 of these $\rightarrow$ $(\vec{v}, \lambda \vec{v})$ for $\vec{v} \neq \vec{0}$ and $\lambda \in \mathbb{F}_3$

1 of these $\rightarrow$ $(\vec{0}, \vec{v})$ For $\vec{v} \neq 0$.

$g \cdot (\vec{v}, \lambda \vec{v}) = (g\vec{v}, \lambda g \vec{v})$

SA 4:

How many elements are in the conjugacy class of $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \in GL_2(\mathbb{F}_5)$?

30.

Linear algebra answer: M conjugate to $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\iff$ M has eigenvalues 1, & -1 each w/ multiplicity 1.

To give M — just need to give the 1-dim'l subspaces $V_1 \subseteq \mathbb{F}_5^2$, $V_{-1} \subseteq \mathbb{F}_5^2$ s.t. $Mv = v$ for $v \in V_1$ & $Mv = -v$ for $v \in V_{-1}$.

$V_1 \neq V_{-1}$.

$\underset{5+1}{6}$ 1-dim'l subspaces of $\mathbb{F}_5^2$

$\implies$ 6 choices for V_1 $\times$ 5 choices for V_{-1}

$= 30.$

Group theory reason: $\left| \text{orbit of } \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ under conjugation} \right| = \dfrac{|GL_2(\mathbb{F}_5)|}{\left|\text{stab} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right|}$

$$\text{stab} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} * & 0 \\ 0 & * \end{bmatrix} \quad * \in \mathbb{F}_5^\times$$

has order $4^2 = 16$.

$$|GL_2(\mathbb{F}_5)| = (5^2-1)(5^2-5)$$
$$= 4 \cdot 6 \cdot 5 \cdot 4$$

$6 \cdot 5$
$\|$
30

1. Every group of order $121 = 11^2$ is abelian.

TRUE. (In class we saw this). group of order p^2

2. Every group of order $57 = 3 \cdot 19$ is abelian

FALSE. $\mathbb{Z}/19\mathbb{Z} \rtimes \mathbb{Z}/3\mathbb{Z}$

Take $\mathbb{Z}/3\mathbb{Z} \to (\mathbb{Z}/19\mathbb{Z})^\times \cong \mathbb{Z}/18\mathbb{Z}$

non-trivial map.

3. Every group of order $91 = 7 \cdot 13$ is cyclic.

TRUE $\quad n_{13} = 1 \quad n_7 = 1.$

4. Every subgroup of Q_8 is normal

TRUE. (In an exercise).

5. The dihedral group D_{30} is solveable.

TRUE $\sim$ rotations are a normal subgroup of index 2.

6. There is a simple group of order 60

TRUE : A_5.

7. There is a simple group of order 168.

TRUE: $GL_3(\mathbb{F}_2)$. (in class).

8. There is a simple group of order $88 = 8 \cdot 11$

FALSE: $n_{11} = 1$.

9. There is a simple group of order $360 = 2^3 \cdot 3^2 \cdot 5$.

TRUE: A_6.

10. There is a simple group of order $256 = 2^8$.

FALSE: Groups prime power order are solveable.

SA

1. How many Sylow 7-subgroups in a simple group of order $168 = 2^3 \cdot 3 \cdot 7$.

$n_7 = 1$ or 8

but can't be 1 because simple.

So $\boxed{8}$

2. How many conjugacy classes in S_5? 7

{e} (12)(34)

2-cycles 2×2-cycles

3-cycles A 2-cycle + 3-cycle (12)(345).

4-cycles

5-cycles

3. How many elements are in the center of D_{14}?

1. $Z(D_{14}) = \{e\}$.

180° rotation isn't a symmetry of a regular n-gon when n is odd.