

- Midterm on Thursday:
  - 10 T/F questions.
  - 5 very-short answer questions
  - a positive integer.
  - 1 hour.
  - + extra time to upload.
- Office hours Tuesday (tonight) at 8:30pm - 10:00pm
- Next week Class optional
  - Tuesday 3/9 go over the midterm.
  - Thursday 3/11 class = office hours
  - office hours = don't exist.

## Exam:

From the syllabus / bulletin:

Examples  
of group  
+ action

Subgroups, normal subgroups, quotient groups,  
homomorphisms, isomorphism theorems,  
groups acting on sets, orbits, stabilizers,  
orbit decomp. formula, Lagrange's Theorem,  
(structure of  $G$ -sets).

Cayley's Theorem, Sylow Theorems,  
permutation groups, symmetric & alternating groups,  
simplicity of  $A_n, n \geq 5$ , direct & semidirect  
products, exact & split exact sequences,  
commutator subgroup, soluble groups, nilpotent groups

solvability of  $p$ -groups, classical matrix groups,  
 (nilpotent)  
 automorphism groups, Jordan-Hölder theorem,  
~~free groups, presentations of groups~~

Week 4, Exercise 4 - (1).

Suppose  $K$  is a cyclic group  
 ( $\mathbb{Z}/n\mathbb{Z}$  or  $\mathbb{Z}$ )

and  $\phi_i: K \rightarrow \text{Aut}(H)$  ( $H$  a group).

Suppose the subgroups  $\phi_1(K) \leq \phi_2(K)$

are conjugate in  $\text{Aut}(H)$ .

(If  $K$  is infinite: assume both are injective).

Construct an explicit isomorphism

$$H \rtimes_{\phi_1} K \cong H \rtimes_{\phi_2} K.$$

$$\mathbb{Z} \rightarrow (\mathbb{Z}/11\mathbb{Z})^\times = \text{Aut}(\mathbb{Z}/11\mathbb{Z})$$

$$\phi_1: 1 \mapsto 2$$

$$\phi_2: 1 \mapsto 6$$

$$5^2 = 24$$

$$\begin{array}{cccc}
 & 3^3 & 5^2 & 5 \\
 & \nearrow & \nearrow & \nearrow \\
 & 15 & 20 & 45 \\
 & \parallel & \parallel & \parallel \\
 & 3 & 4 & 1
 \end{array}$$

~~Can't precompose  
 with an elt of  $\mathbb{Z}$   
 to get from  $\phi_1$  to  $\phi_2$ .~~

$$6^2 = 36$$

$$\begin{array}{c}
 \parallel \\
 3
 \end{array}$$

$$\text{Aut}(\mathbb{Z}) = \{\pm 1\} (\cong \mathbb{Z}^\times).$$

$$\left( \phi_1 \circ (-1) \right) (1) = \phi_1(-1) = 2^{-1} = 6.$$

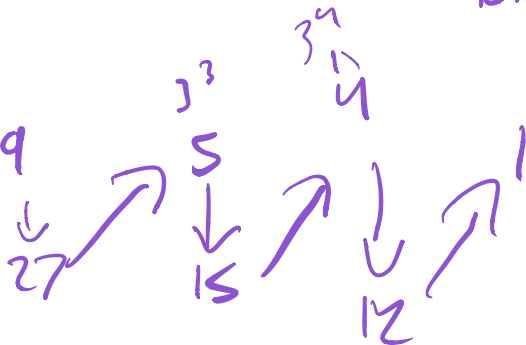
$$\phi_1 \circ (-1) = 2.$$

Can use this to show  $\mathbb{Z}/12\mathbb{Z} \rtimes_{\phi_1} \mathbb{Z} \cong \mathbb{Z}/12\mathbb{Z} \rtimes_{\phi_2} \mathbb{Z}$

$\phi(10) = 4$ .  $\leftarrow$  # of generators of  $(\mathbb{Z}/12\mathbb{Z})^\times \cong \mathbb{Z}/6\mathbb{Z}$ .

$$3 \in (\mathbb{Z}/12\mathbb{Z})^\times$$

$$3^2 = 9$$



$$3^5 = 1$$

$-3 \in (\mathbb{Z}/12\mathbb{Z})^\times$  has order 10

$$\downarrow$$

$$\mathbb{Z} (\mathbb{Z}/12\mathbb{Z})^\times$$

should take  $\phi_1: 1 \mapsto 2$

$$\phi_2^1: 1 \mapsto 8$$