

6320 - Modern Alg II.

Groups, Galois theory, Representation theory.

Rough schedule: \uparrow Field extensions.

Week 1 - Motivation, first concepts

Weeks 2-5 - The structure of finite groups.

Week 6 - Character theory of abelian group
 $\uparrow \text{Hom}(G, \mathbb{C}^*)$.

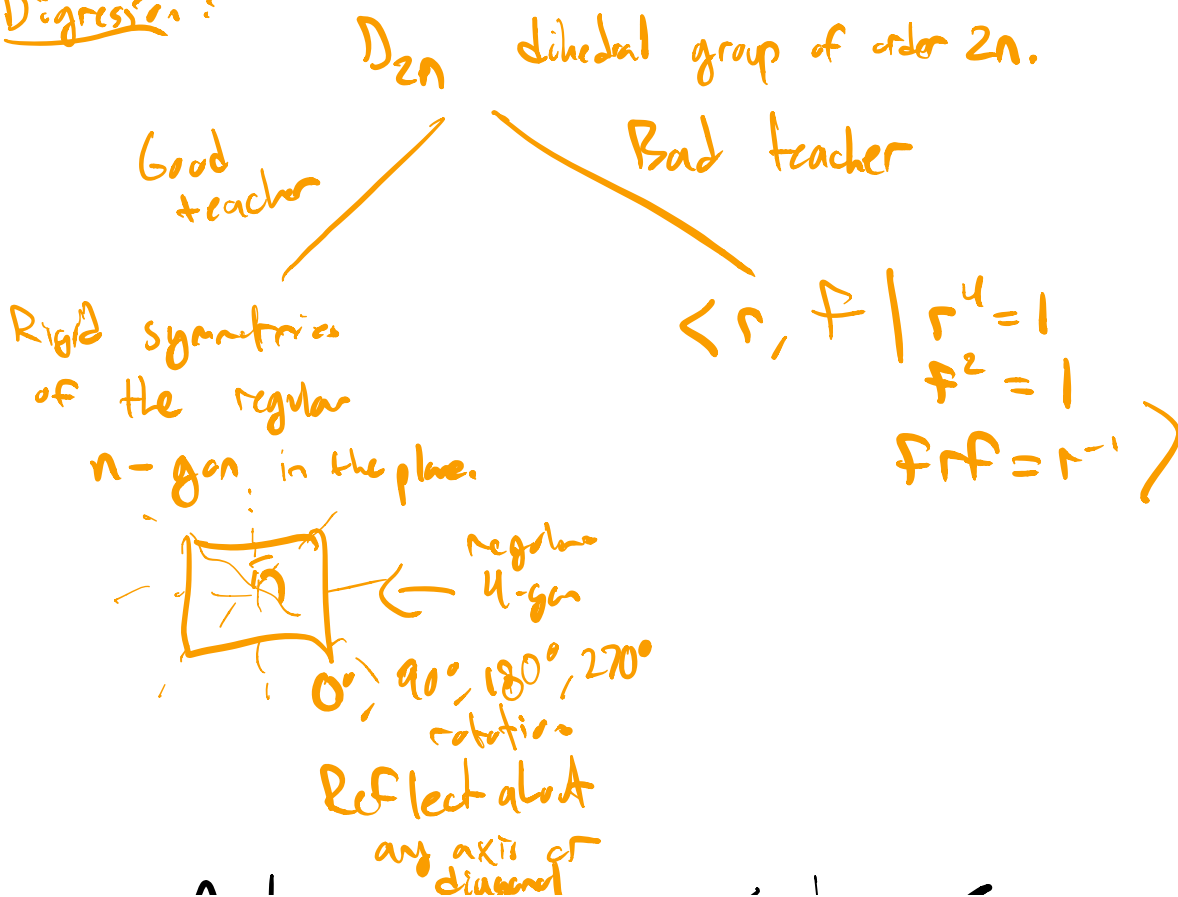
Week 7 - Review, midterm.

Week 8 - Spring Pause.

Weeks 9-12 - Galois theory.

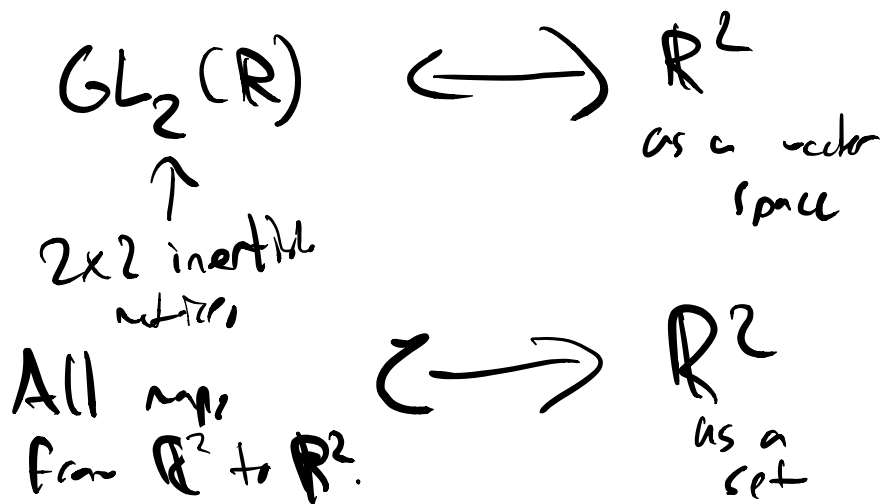
Weeks 13-15 - Repr'n theory of finite groups.

Digression:



Automorphisms preserving structure = Set w/
 Symmetries \longleftrightarrow Structure.

Fewer symmetries. = $\begin{cases} \text{More structure} \end{cases}$



Def'n A group is a set G equipped with

$$m: G \times G \rightarrow G$$

$$(a, b) \mapsto ab$$

$$i: G \rightarrow G$$

$$a \mapsto a^{-1}$$

$$e: \{*\} \rightarrow G$$

$$* \mapsto e.$$

satisfy some compatibilities:
 associativity, existence of inverses

" " " "

↓ e.g.

$$aa^{-1} = e$$

$$m(a, m(a)) = e.$$

Map of groups = maps of sets
(homomorphisms). respecting extra structures

$$f(ab) = f(a)f(b)$$

$$f(a^{-1}) = f(a)^{-1}$$

$$f(e) = e.$$

Examples: $\mathbb{Z}/n\mathbb{Z}$
 ↑
 cyclic group of order n

$GL_n(\mathbb{R})$
 ↑
 General linear group over \mathbb{R}

S_n
 ↑
 Symmetric group on n-elts

$U(n)$ $SL_n(\mathbb{R})$

Aut cov. $(\tilde{X} \rightarrow X)$
 ↓ spec. ↓ univ. cov. (if U_1)

$$\mathbb{R}^X = GL_1(\mathbb{R})$$

$\pi_1(X, x) \leftarrow$ Fundamental group.

$D_{2n} \leftarrow$ Dihedral group.

$Diff / Diff^0 \leftarrow$ Mapping class group.

R^\times For R any ring.
 = units in R .

$$S_n = \text{Aut}_{\text{set}} (\{1, 2, \dots, n\})$$

$$GL_n(\mathbb{R}) = \text{Aut}_{\mathbb{R}\text{-vector space}} (\mathbb{R}^n)$$

$(\mathbb{R}$ commutative) $\hookrightarrow GL_n(\mathbb{R}) = \text{Aut}_{\mathbb{R}\text{-module}} (\mathbb{R}^n)$

$$GL_1(\mathbb{R}) = \text{Aut}_{\mathbb{R}\text{-module}} (\mathbb{R})$$

\parallel
 \mathbb{R}^\times

$$O(n) = \text{Aut}_{\text{inner product spaces}} (\mathbb{R}^n, \langle \cdot, \cdot \rangle)$$

$\hat{=}$ standard inner product.

$$SL_n(\mathbb{R}) = \text{Aut}_{\text{vector spaces with volume + orientation}} (\mathbb{R}^n, e_1^* \wedge e_2^* \wedge \dots \wedge e_n^*)$$

\uparrow
basis for $\wedge^n(\mathbb{R}^n)^\times$

$$= \text{Aut}_{\mathbb{R}\text{-vector spaces with volume + orientation}} (\mathbb{R}^n)$$

$$S^1 = \{z \mid |z|=1\}$$

$$\mathbb{R} \xrightarrow{\pi} S^1$$
$$t \mapsto e^{2\pi i t}$$

$$\mathcal{Z} = \text{Aut}(\mathbb{R} \rightarrow S^1)$$

\uparrow acting by translations

$\overset{\text{is}}{\parallel}$ Aut of topological space \mathbb{R}
sit. $\pi(\sigma(t)) = \pi(t)$.

$$n = t \mapsto t + n$$

G a group.

G is a right G -set.

$$x \cdot g = xg$$

$$\text{Aut}_{\text{Right } G\text{-set}}(G) = G$$

\hookrightarrow acting by multiplication on the left.