Exam II - Formula Sheet

1. \( \frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1 - x^2}} \)

2. \( \frac{d}{dx} \arctan(x) = \frac{1}{1 + x^2} \)

3. \( \sum_{k=1}^{\infty} ar^{k-1} = \sum_{k=0}^{\infty} ar^k = \frac{a}{1 - r} \)

4. The Taylor series for \( f(x) \) is given by

\[
   f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \cdots
\]

**Theorem 1 (Integral Test).** Let \( f \) be a continuous, positive, nonincreasing function on the interval \([1, \infty)\) and suppose that \( a_k = f(k) \) for all positive integers \( k \). Then the infinite series

\[
   \sum_{k=1}^{\infty} a_k
\]

converges if and only if the improper integral

\[
   \int_{1}^{\infty} f(x) \, dx
\]

converges.

**Theorem 2 (Alternating Series Test).** Let

\[
   a_1 - a_2 + a_3 - a_4 + \cdots
\]

be an alternating series with \( a_n > a_{n+1} > 0 \). If \( \lim_{n \to \infty} a_n = 0 \), then the series converges.

**Theorem 3 (Limit Comparison Test).** Suppose that \( a_n \geq 0, b_n > 0 \), and

\[
   \lim_{n \to \infty} \frac{a_n}{b_n} = L.
\]

If \( 0 < L < \infty \), then \( \Sigma a_n \) and \( \Sigma b_n \) converge or diverge together. If \( L = 0 \) and \( \Sigma b_n \) converges, then \( \Sigma a_n \) converges.
Theorem 4 (Absolute Ratio Test). Let $\sum u_n$ be a series of nonzero terms and suppose that

$$\lim_{n \to \infty} \frac{|u_{n+1}|}{|u_n|} = \rho$$

If $\rho < 1$, the series converges absolutely. If $\rho > 1$, the series diverges. If $\rho = 1$, the test is inconclusive.