

Exam II - Formula Sheet

1. $\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$
2. $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$
3. $\sum_{k=1}^{\infty} ar^{k-1} = \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$
4. The Taylor series for $f(x)$ is given by

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \dots$$

Theorem 1 (Integral Test). Let f be a continuous, positive, nonincreasing function on the interval $[1, \infty)$ and suppose that $a_k = f(k)$ for all positive integers k . Then the infinite series

$$\sum_{k=1}^{\infty} a_k$$

converges if and only if the improper integral

$$\int_1^{\infty} f(x) dx$$

converges.

Theorem 2 (Alternating Series Test). Let

$$a_1 - a_2 + a_3 - a_4 + \dots$$

be an alternating series with $a_n > a_{n+1} > 0$. If $\lim_{n \rightarrow \infty} a_n = 0$, then the series converges.

Theorem 3 (Limit Comparison Test). Suppose that $a_n \geq 0, b_n > 0$, and

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L.$$

If $0 < L < \infty$, then $\sum a_n$ and $\sum b_n$ converge or diverge together. If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

Theorem 4 (Absolute Ratio Test). *Let $\sum u_n$ be a series of nonzero terms and suppose that*

$$\lim_{n \rightarrow \infty} \frac{|u_{n+1}|}{|u_n|} = \rho$$

If $\rho < 1$, the series converges absolutely. If $\rho > 1$, the series diverges. If $\rho = 1$, the test is inconclusive.