Final Exam - Formula Sheet

1. $\displaystyle\int \frac{1}{u^2+a^2}\,du = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$

2. $a+ar+ar^2+ar^3+\cdots$ *converges to* $\dfrac{a}{1-r}$ *when* r *satisfies appropriate conditions.*

3. *The Taylor series for* $f(x)$ *centered at* a *is given by*

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \cdots$$

4. *Integration by parts:*

$$\int u\,dv = uv - \int v\,du.$$

5. *Area inside a polar curve:*

$$\int_\alpha^\beta \frac{1}{2}[f(\theta)]^2\,d\theta.$$

Theorem 1 (Integral Test). *Let* f *be a continous, positive, nonincreasing function on the interval* $[1,\infty)$ *and suppose that* $a_k = f(k)$ *for all positive integers* k. *Then the infinite series*

$$\sum_{k=1}^{\infty} a_k$$

converges if and only if the improper integral

$$\int_1^\infty f(x)\,dx$$

converges.

Theorem 2 (Alternating Series Test). *Let*

$$a_1 - a_2 + a_3 - a_4 + \cdots$$

be an alternating series with $a_n > a_{n+1} > 0$. *If* $\lim_{n\to\infty} a_n = 0$, *then the series converges.*

Theorem 3 (Limit Comparison Test). *Suppose that $a_n \geq 0, b_n > 0$, and*

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L.$$

If $0 < L < \infty$, then Σa_n and Σb_n converge or diverge together. If $L = 0$ and Σb_n converges, then Σa_n converges.

Theorem 4 (Absolute Ratio Test). *Let Σu_n be a series of nonzero terms and suppose that*

$$\lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|} = \rho$$

If $\rho < 1$, the series converges absolutely. If $\rho > 1$, the series diverges. If $\rho = 1$, the test is inconclusive.