

1. 5 points. Find the inverse of

$$f(x) = \frac{5x + 6}{3x + 2}.$$

Solution.

$$\begin{aligned}y &= \frac{5x + 6}{3x + 2} \\x &= \frac{5y + 6}{3y + 2} \\3xy + 2x &= 5y + 6 \\3xy - 5y &= 6 - 2x \\y(3x - 5) &= 6 - 2x \\y &= \frac{6 - 2x}{3x - 5}.\end{aligned}$$

2. 3 points. Let $f(x) = x^6 + 7x^5 - x^4 - 62x^3 - 30x^2 + 26x + 6$. Compute $f(-5)$ by using synthetic division.

Solution. Carrying out synthetic division with the root -5 gives a remainder of 1. To get full credit, you needed to specify somewhere that $f(-5)$ is equal to 1.

3. 7 points Let $f(x) = 2x^4 - 11x^3 + 20x^2 - 7x - 10$. Given that 2 and $-1/2$ are roots of f , find all the roots of f . Simplify the roots by pulling squares out of square roots and writing $\sqrt{-1}$ as i . *Hint: you will need to use the quadratic formula to find the last two roots of f , i.e.*

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Solution. We first perform synthetic division with the root 2 and get the polynomial $2x^3 - 7x^2 + 6x + 5$. Carrying out synthetic division with the root $-1/2$ then gives the polynomial $2x^2 - 8x + 10$. To simplify subsequent computations, we can divide this polynomial by 2 and obtain $x^2 - 4x + 5$. Applying the quadratic formula to this last polynomial and simplifying gives us the roots $2 \pm i$.

4. Let $f(x) = 2x^4 + 5x^3 + 5x^2 + 20x - 12$.

(1) 2 point. Given that $f(-2i) = 0$, what other number must be a root of f ?

By conjugate pairing, $2i$ is also a root.

(2) 3 points. Let $a = -2i$ and let b be the root found in the previous part. Compute $(x-a)(x-b)$. After simplifying, you should have a quadratic polynomial with integer coefficients. Call this polynomial g .

$$(x + 2i)(x - 2i) = x^2 - (2i)^2 = x^2 + 4.$$

(3) 4 points. Define $h(x) = f(x) \div g(x)$. Compute h . You should again get a quadratic polynomial with integer coefficients.

$$h(x) = 2x^2 + 5x - 3.$$

(4) 4 points. Find the roots of $h(x)$.

$$\begin{aligned}2x^2 + 5x - 3 &= (2x - 1)(x + 3) = 0. \\x &= 1/2, -3.\end{aligned}$$

5. 7 points. Find the vertical and horizontal asymptotes of

$$f(x) = \frac{3x^2 + 7x + 2}{2x^2 + x - 3}.$$

Show work justifying your answer.

Solution. The numerator and denominator have the same degree in this case, so the horizontal asymptote is found by taking the quotient of the coefficients, which is $3/2$.

To find the vertical asymptotes, we factor the denominator:

$$2x^2 + x - 3 = (2x + 3)(x - 1).$$

Thus, we potentially have vertical asymptotes at $-3/2$ and 1 . We check that these values are not roots of the numerator and see that they are in fact vertical asymptotes.

6. Let $f(x) = 2x^3 + x^2 - 7x - 6$.

(1) 5 points. According to the rational root theorem, what are the possible rational roots of f ?

$$p = 1, 2, 3, 6.$$

$$q = 1, 2.$$

Possible roots: $\pm 1 \pm 2, \pm 3, \pm 6, \pm 1/2, \pm 3/2$.

(2) 4 points. Find a rational root of $f(x)$ that is greater than 1 and less than 3.

Possible rational roots in the specified range are $3/2$ and 2 . Testing each of these by synthetic division, we find that 2 is a root and that division by $x - 2$ gives the polynomial $2x^2 + 5x + 3$.

(3) 5 points. Find the other roots of f .

$$2x^2 + 5x + 3 = (2x + 3)(x + 1).$$

$$x = -3/2, -1.$$

7. 5 points. Rewrite $\frac{3 - 2i}{5 + i}$ in the form $a + bi$. You will get fractions for a and b .

$$\frac{3 - 2i}{5 + i} = \frac{3 - 2i}{5 + i} \cdot \frac{5 - i}{5 - i} = \frac{15 - 3i - 10i - 2}{25 + 1} = \frac{13 - 13i}{26} = \frac{1}{2} - \frac{1}{2}i.$$

8. 3 points. Rewrite $\log_5 7$ in terms of logarithms with base 3.

$$\frac{\log_3 7}{\log_3 5}.$$

9. 7 points Use the properties of the logarithm to expand the following as a sum, difference and/or constant multiple of logarithms.

$$\ln \left(\frac{7x^2}{(x-2)(x+3)} \right), \quad x > 2.$$

$$\ln \left(\frac{7x^2}{(x-2)(x+3)} \right) = \ln(7x^2) - \ln((x-2)(x+3)) = \ln 7 + \ln x^2 - (\ln(x-2) + \ln(x+3)) = \ln 7 + 2 \ln x - \ln(x-2) - \ln(x+3).$$

10. 7 points. Solve the following exponential equation. Remember to check for extraneous solutions.

$$3e^{2x} - 3e^x - 10 = 2e^{2x}.$$

Solution. Let $y = e^x$. We then convert the above equation to

$$3y^2 - 3y - 10 = 2y^2.$$

$$y^2 - 3y - 10 = 0.$$

$$(y - 5)(y + 2) = 0.$$

$$y = 5, -2.$$

$$e^x = 5, -2$$

The second solution is extraneous since e^x always gives a positive number. The first solution gives us $x = \ln 5$.

11. 7 points. Solve the following logarithmic equation. Remember to check for extraneous solutions.

$$\log_7 x + \log_7(5x - 2) = \log_7(x + 2).$$

$$\log_7 x(5x - 2) = \log_7(x + 2).$$

$$x(5x - 2) = x + 2.$$

$$5x^2 - 2x = x + 2.$$

$$5x^2 - 3x - 2 = 0.$$

$$(5x + 2)(x - 1) = 0.$$

$$x = -2/5, 1.$$

The first solution is extraneous since $\log_7(-2/5)$ is undefined.

12. 2 points. Simplify $\log_3 81$.

This expression is equal to 4 because $3^4 = 81$.