1. Solve the system

$$x^2 + 4y - x = 8$$
$$x + y = 3.$$

Check your work by substituting your answer in the original system.

solution.

$$(x, y) = (1, 2) \text{ and } (4, -1).$$

□

2. At a grocery store, mozzarella cheese is sold in half pound blocks for \$2 each. One pound blocks of cheddar cheese cost \$3 each. If a shopper spent \$13 on cheddar and mozzarella cheese to buy 4 pounds of cheese, how many blocks of cheddar and mozzarella cheese were purchased?

solution. Two blocks of mozzarella and three blocks of cheddar. □

3. Solve the system

$$x + 2y - z = 10$$
$$2y + z = 2$$
$$2x + 5y - 5z = 28.$$

Check your work by substitution.

solution.

$$(x, y, z) = (4, 2, -2).$$

□

4. Rewrite the given matrix as a system of equations with variables x, y, z and w.

$$\left[\begin{array}{cccc|c} 1 & 5 & -2 & 3 & 4 \\ 3 & 2 & 5 & 1 & 11 \\ 0 & 6 & 11 & -1 & 2 \\ 7 & 2 & 2 & 0 & -1 \end{array}\right]$$

5.

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \\ 0 & -2 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 5 & 0 \end{bmatrix}$$

Compute AB and BA.

solution.

$$AB = \begin{bmatrix} 2 & 9 & -2 \\ 6 & 23 & -4 \\ -4 & -10 & 0 \end{bmatrix}$$
$$BA = \begin{bmatrix} 8 & 8 \\ 24 & 17 \end{bmatrix}.$$

□

6. Find the inverse of

$$A = \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

Verify your answer by computing A times A^{-1}.

solution.

$$A^{-1} = \begin{bmatrix} -2 & 5 & 3 \\ 1 & -2 & -1 \\ -1 & 3 & 1 \end{bmatrix}$$

□

7. Simplify

$$3\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}.$$

solution.

$$\begin{bmatrix} 5 & 9 \\ 4 & -8 \end{bmatrix}$$

□

8. Compute the determinant of

$$\begin{bmatrix} 3 & 0 & -1 \\ 1 & 5 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$

solution. 8 □

9. Compute

$$\sum_{i=1}^{7} (-1)^i$$

solution. -1 □

10. For an arithmetic sequence, suppose that a_5=13 and $a_{11} = 37$. Compute a_{20}.

solution.

$$a_n = 4n - 7.$$

$$a_{20} = 73.$$

□

11. Simplify $\frac{11!}{8!}$.

solution.

$$11 \cdot 10 \cdot 9 = 990.$$

□

12. Let $3, 7, 11, 15$ be the first four terms of an arithmetic sequence. Write down the general formula for a_n.

solution.

$$a_n = 4n - 1.$$

□

13. Given an arithmetic sequence with $a_6 = 5$ and $a_{13} = 47$, find S_{20}, the sum of the first twenty terms of this sequence.

solution.

$$a_n = 6n - 31.$$
$$a_1 = -25.$$
$$a_{20} = 89.$$
$$S_{20} = 10(64) = 640.$$

□

14. Compute

$$\sum_{i=1}^{5} 2^{i-1}$$

solution. 31 □

15. Suppose that a sequence is defined recursively as follows: $a_1 = 2$, $a_2 = -1$ and $a_{k+2} = a_k + a_{k+1}$. Write down the first five terms of this sequence.

solution.

$$a_1 = 2, a_2 = -1, a_3 = 1, a_4 = 0, a_5 = 1.$$

□

16. Write an expression for the nth term of the sequence:

$$-\frac{2}{5}, \frac{2}{25}, -\frac{2}{125}, \frac{2}{625}, \ldots$$

solution.

$$a_n = 2\left(-\frac{1}{5}\right)^n.$$

□

A few more examples of inverses:

17.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$
$$A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & -1 \\ -2 & 4 & -1 \end{bmatrix}$$

18.

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$
$$A^{-1} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ -1 & 2 & 1 \end{bmatrix}$$

19.

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 7 & -2 \\ 1 & 6 & -2 \end{bmatrix}$$
$$A^{-1} = \begin{bmatrix} 2 & -10 & 11 \\ 0 & 1 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

20.

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 2 \end{bmatrix}$$