

MATH 1050 SECTION 2
FALL 2009
PRACTICE EXAM I SOLUTIONS

1. GENERAL GUIDELINES

1.1. **Seating.** Seating for this exam will be assigned. I will begin passing out exams at 9:30. Your assigned seat will be written on the front of your exam. In addition, please bring your ID. I will check it as you turn in your exam.

1.2. **Calculators.** You will not be permitted to use calculators. As such, the problems will involve relatively simple arithmetic that you should be able to do by hand.

1.3. **Material Covered.** The exam will cover material through section 1.8 in the book. You should know how to do all questions from the homework.

1.4. **Showing Work.** You should show enough work so that the graders can follow your thought process. When in doubt, show your work. I also highly recommend that you check your answer for consistency where possible. For example, if you are asked to find the equation of a line passing through two points, you should check that the two points satisfy the equation you've obtained.

1.5. **Studying.** Mathematics is more a set of skills than a collection of facts. Practice is the key to mastering these skills. As such, I recommend that you work as many problems as possible rather than simply memorizing formulas from the book. Work the problems below and then rework as many homework problems as you can. If you have time, work extra problems from the book.

2. SOLUTIONS

- (1) Simplify the expression $\sqrt{54xy^4}$. Make sure to put in absolute value bars where they are needed.

Solution.

$$\sqrt{54xy^4} = \sqrt{3^3 \cdot 2} \sqrt{x} \sqrt{y^4} = 3\sqrt{6} \sqrt{x} |y^2|.$$

Notice that the absolute value bars aren't strictly necessary here since y^2 is already non-negative. Your answer would be correct with or without the absolute value. \square

- (2) Multiply out $(x + 3)(x + 4)$ and simplify. For which values of x should this expression go to 0? Plug these values into the simplified expression to verify your work.

Solution. Using FOIL, we get

$$x^2 + 4x + 3x + 12 = x^2 + 7x + 12.$$

We can see that the roots of the expression are -3 and -4 by looking at the factors. To check, we simply plug these numbers into the multiplied expression:

$$(-3)^2 + 7(-3) + 12 = 9 - 21 + 12 = 0.$$

$$(-4)^2 + 7(-4) + 12 = 16 - 28 + 12 = 0.$$

\square

- (3) Factor $x^2 + 10x + 25$. *Hint: perfect square.* For which values x should this polynomial go to 0? Verify your work by plugging these values into the original expression.

Solution. Using the formula for perfect squares, we get $(x+5)^2$. (You can use another method of factorization if you wish.) We can see that the expression goes to 0 when $x = -5$. We check our factorization by plugging into the original formula:

$$(-5)^2 + 10(-5) + 25 = 25 - 50 + 25 = 0.$$

□

- (4) Factor $12x^2 - 48$. *Hint: difference of two squares.* Make sure to pull out integer factors.

Solution. We'll start by pulling out integer factors to get $12(x^2 - 4)$. Then, using the formula for a difference of two squares, we obtain $12(x - 2)(x + 2)$. □

- (5) Factor $4x(2x - 1) + (2x - 1)^2$ by grouping. Make sure that each of your linear factors (with form $ax + b$) is completely simplified.

Solution. We are told to factor by grouping, but in fact the grouping is already done for us. We need only pull out common factors:

$$4x(2x - 1) + (2x - 1)(2x - 1) = (2x - 1)(4x + 2x - 1) = (2x - 1)(6x - 1).$$

□

- (6) Factor $2x^2 + 9x + 9$.

Solution. You can use your favorite method here. We will factor by trial and error. If this polynomial has a factorization, then it can be written in the form $(2x + a)(x + b)$, where $ab = 9$. After trying a few values for a and b , we get $(2x + 3)(x + 3)$. □

- (7) Use the quadratic formula to solve the equation

$$x^2 - 10x + 22 = 0.$$

Solution. $a = 1$, $b = -10$ and $c = 22$. The quadratic formula then gives us

$$\frac{10 \pm \sqrt{100 - 4 \cdot 1 \cdot 22}}{2} = \frac{10 \pm \sqrt{12}}{2} = \frac{10 \pm 2\sqrt{3}}{2} = 5 \pm \sqrt{3}.$$

□

- (8) Start with the expression

$$\frac{x^2 + 8x - 20}{x^2 + 11x + 10}.$$

Factor the numerator and denominator completely. Simplify by cancelling common factors. What point must be removed from the domain of the new expression to account for the cancelled factors?

Solution.

$$\frac{x^2 + 8x - 20}{x^2 + 11x + 10} = \frac{(x - 2)(x + 10)}{(x + 1)(x + 10)} = \frac{x - 2}{x + 1}.$$

Because we cancelled out $(x + 10)$, the new expression is defined at $x = -10$ while the original one was not, so we must specify that the domain is $x \neq -10$. □

- (9) Find all solutions to the equation $|3x + 2| = 7$.

Solution. In this equation, either $3x + 2 = 7$ or $-(3x + 2) = 7$. We solve each of these separately to obtain the solutions $5/3$ and -3 . Plug these back into the original equation to check your work. □

- (10) Find all solutions to the equation $\frac{2}{(x - 4)(x - 2)} = \frac{1}{x - 4} + \frac{2}{x - 2}$. Take special care to show your thought process in this question.

Solution. First, we multiply by $(x - 4)(x - 2)$ to cancel the denominators. This gives us the equation

$$2 = 1 \cdot (x - 2) + 2 \cdot (x - 4).$$

Simplify and solve:

$$2 = x - 2 + 2x - 8.$$

$$2 = 3x - 10.$$

$$12 = 3x.$$

$$x = 4.$$

We then plug this into the original equation and find that it is not a solution. (We get division by 0.) Thus, the equation has no solutions. \square

- (11) Find the solution sets to the inequalities $|x + 4| > 2$ and $|x - 3| < 4$.

Solution. The first inequality looks like $|x| > a$. This type splits into two solution sets:

$$x + 4 < -2 \text{ or } x + 4 > 2.$$

Solving each of these, we have the solutions sets $x < -6$ and $x > -2$. The second inequality looks like $|x| < a$. This type can be rewritten as

$$-4 < x - 3 < 4.$$

Adding 3 to each part of the inequality gives us

$$-1 < x < 7.$$

See page A64 in your book for more review on this subject. \square

- (12) Find the distance between the points $(3, 11)$ and $(2, 4)$. Find the midpoint of the line segment connecting these points.

Solution. Here, we simply apply the formulas:

$$d = \sqrt{(3 - 2)^2 + (11 - 4)^2} = \sqrt{1 + 49} = \sqrt{50} = \sqrt{2 \cdot 25} = 5\sqrt{2}.$$

Midpoint:

$$\left(\frac{3 + 2}{2}, \frac{11 + 4}{2} \right) = \left(\frac{5}{2}, \frac{15}{2} \right).$$

\square

- (13) Write an equation for the circle with center at $(2, 3)$ and radius 3. Graph this circle.

Solution.

$$(x - 2)^2 + (y - 3)^2 = 9.$$

\square

- (14) Find the equation of the line passing through $(4, 6)$ and $(9, 2)$. Find the slope of this line. Using the equation you wrote down, find the y -intercept for this line. Use your calculations to write the slope intercept form of the equation of this line. Verify both equations (the two point form and the point-slope form) by checking that the points satisfy the equations.

Solution. We'll first compute the slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{2 - 6}{9 - 4} = -\frac{4}{5}.$$

Now we use the point slope form of the equation of a line:

$$(y - 6) = -\frac{4}{5}(x - 4).$$

To find the y -intercept, we can set x equal to 0:

$$(y - 6) = -\frac{4}{5}(-4).$$

$$y - 6 = \frac{16}{5}.$$

$$y = \frac{46}{5}.$$

□

- (15) Return to the graph of the circle above. Does this graph come from a function? *Hint: vertical line test.* Explain how the test shows whether or not the graph is from a function. What property of a function does the test check for?

Solution. Applying the vertical line test shows that the circle does not come from a function. The vertical line test shows us that the circle has more than one y -value corresponding to each x -value, where a function sends each x -value in its domain to exactly one y -value. □

- (16) Does the following equation represent y as a function of x ?

$$|y| = 4 - x.$$

Solution. No. For example, $x = 0$ corresponds to both $y = 4$ and $y = -4$. □

- (17) Find the average rate of change of the function $x^2 + 3x + 4$ from $x = 3$ to $x = 7$. Sketch the function. Draw a line connecting the points on the graph at $x = 3$ and $x = 7$. How does this line relate to the average rate of change?

Solution.

$$\frac{\Delta f}{\Delta x} = \frac{f(7) - f(3)}{7 - 3} = \frac{74 - 22}{4} = \frac{52}{4} = 13.$$

The slope of the line on the graph is equal to the average rate of change you just calculated. □

- (18) Is the function $f(x) = x^6 - 2x^2 + 3$ even or odd? If you used the shortcut method to determine this, check using the formal definition. What symmetry does the graph have?

Solution.

$$f(-x) = (-x)^6 - 2(-x)^2 + 3 = x^6 - 2x^2 + 3 = f(x).$$

It follows that f is even. Thus, it is symmetric under reflection across the y -axis. □

- (19) Start with the function $f(x) = x^3$. Move it six units to the left, six units downward and reflect across the y -axis. Write down the expression for the new function $h(x)$.

Solution. Six units to the left:

$$(x + 6)^3.$$

Six units downward:

$$(x + 6)^3 - 6.$$

Reflect across the y -axis:

$$h(x) = (-x + 6)^3 - 6.$$

□

- (20) Let $g(x) = |3 - x| - 8$. What is the basic function type? Write down the series of geometric transformations that takes you from the basic function to $g(x)$. Check your work by applying these transformations to the basic function to get $g(x)$.

Solution. Basic function type: $|x|$. To find the series of transformations, we work backwards:
Undo shift 8 units down:

$$|3 - x|.$$

Undo reflection across the y -axis:

$$|3 + x|.$$

Undo shift 3 units to the left:

$$|x|.$$

So the series of transformations is: shift 3 units to the left, reflect across the y -axis, shift 8 units down. \square

- (21) Let $g(x) = 5x^2$ and let $h(x) = x^3 + 4$. Compute $g \circ h(x)$ and $h \circ g(x)$. Compute $g - h$ and $g + h$.

Solution.

$$\begin{aligned}g \circ h(x) &= 5(h(x))^2 = 5(x^3 + 4)^2. \\h \circ g(x) &= (g(x))^3 + 4 = (5x^2)^3 + 4. \\(g - h)(x) &= 5x^2 - (x^3 + 4). \\(g + h)(x) &= 5x^2 + x^3 + 4.\end{aligned}$$

\square

- (22) Let $f(x) = \sqrt{24 - x^2}$ and let $g(x) = x - 4$. Compute $\frac{f}{g}$. What is the domain of the quotient?

Solution.

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{24 - x^2}}{x - 4}.$$

First, find the domain of the numerator:

$$24 - x^2 \geq 0.$$

$$24 \geq x^2.$$

$$\sqrt{24} \geq |x|.$$

$$-\sqrt{24} \leq x \leq \sqrt{24}.$$

In addition, we must specify $x \neq 4$ to prevent division by 0. \square