1. Let $f(x) = 6x^3 + 7x^2 - x - 2$. Use the fact that $f(-1) = 0$ to factor f completely.

solution. (2x-1)(3x+2)(x+1). □

2. Find x if $\log_2 x = -5$.

solution.

$$x = 1/32$$

□

3. Find the vertex of the parabola given by $f(x) = 2x^2 + 3x - 4$. (Give the x and y-coordinates.) Graph $f(x)$.

solution.

$$(-3/4, -41/8)$$

□

4. Expand $(2x + y)^4$ with terms ordered from greatest to least powers of x.

solution.

$$16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4.$$

□

5. Expand $(3x - 2y)^3$ with terms ordered from greatest to least powers of x.

solution.

$$27x^3 - 54x^2y + 36xy^2 - 8y^3$$

□

6. Let $a_3 = \dfrac{1}{3}$ and $a_7 = \dfrac{625}{3}$ be terms of a geometric sequence. Find S_5, the sum of the first 5 terms of the sequence.

solution. 781/75 □

7. Find the inverse of

$$f(x) = \frac{2x + 5}{3x - 1}.$$

solution.

$$f^{-1}(x) = \frac{x + 5}{3x - 2}.$$

□

8. A firm's profit function is given by

$$f(x) = 102x - x^2 - 200.$$

where x represents the number of units sold.

(1) For which values of x will the firm break even?
(2) Over what range of numbers of units sold will the firm make a profit?

solution. The firm breaks even if 2 or 100 units are sold. It will make a profit if more than 2 or fewer than 100 units are sold. □

9. Let $f(x) = x - 95 - \dfrac{450}{x - 10}$. Find the roots of f.

solution. 5 and 100. □

10. Let $f(x) = \sqrt{x^2 - 6}$. Find the domain of f.

solution. $x \geq \sqrt{6}$ and $x \leq -\sqrt{6}$. □

11. Let $g(x) = \sqrt{x-7}+2$. What is the basic function type? Find a series of geometric transformations to construct $g(x)$ from the basic function. Check your work by applying these transformations to the basic function.

solution. Basic type: $\sqrt{x}$. Translate 7 units to the right, then 2 units up. □

12. Graph $f(x) = 3(x-2)^2 + 4$.

solution. Use the fact that the vertex is at $(2, 4)$, then plot at least one additional point. □

13. Graph $|x|$.

14. Graph x^3.

15. Graph $\sqrt{x}$.

16. Graph $\dfrac{1}{x}$.

17. Graph $3x + 2y = 7$.

solution. Plot at least two points on the line and draw a line through them. □

18. Graph $5x - 3y = 4$.

19. Find a line perpindicular to $7x + 4y = 3$ with the same y-intercept.

solution.

$$y = \frac{4}{7}x + \frac{3}{4}.$$

□

20. Find a line perpindicular to $3x - y = 11$ passing through $(2, 1)$.

solution.

$$y - 1 = -\frac{1}{3}(x-2).$$

□

21. Find a line parallel to $y - 5x = 6$ that passes through $(-1, 1)$.

solution.

$$y - 1 = 5(x+1).$$

□

22. Find a line with x-intercept 6 and slope 5.

solution.

$$y = 5x - 30.$$

□

23. Find the slope and y-intercept of $3y - 7x = 30$.

solution. $m = \dfrac{7}{3}$ and $b = 10$. □

24. Find a line passing through the points $(2, 1)$ and $(0, 3)$.

solution.

$$y - 3 = -x.$$

□

25. Find a polynomial with roots $3/2, -2$ and 1 and y-intercept 5.

solution.

$$f(x) = \frac{5}{6}(2x-3)(x-1)(x+2).$$

□

26. Find the horizontal and vertical asymptotes of $\dfrac{2x^2+3x-2}{3x^2+x-6}$.

solution. Horizontal: $\dfrac{2}{3}$. Vertical: $\dfrac{-1\pm\sqrt{73}}{6}$ (by the quadratic formula). □

27. Five hundred dollars are placed in an account that collects interest at a yearly rate of 5% compounded quarterly.

(1) How much money is in the account after 3 years? Give an exact expression for your answers in terms of an exponential.
Answer: $500(1.0125)^{12}$.

(2) How much time must pass before the balance is \$3000? Give an exact expression for your answer in terms of logarithms.
Answer: $t = \dfrac{\ln 6}{4\ln(1.0125)}$.

28. Find the determinant of

$$\begin{bmatrix} 2 & 3 & -2 \\ -1 & 7 & 0 \\ 2 & 1 & 4 \end{bmatrix}.$$

solution. 98. □

29. Solve the following system:

$$2x+3y-7z = 20.$$
$$x-y+z = -2.$$
$$x-2z = 4.$$

solution. $x = 2, y = 3$ and $z = -1$. □

30. Write the parabola $y = 3x^2+6x-7$ in standard form, i.e. in the form $y = a(x-h)^2+k$.

solution. $3(x+1)^2-10$. □

31. Find the equation for a parabola with vertex $(3/2, 2)$ and y-intercept 4.

solution.

$$\frac{8}{9}(x-3/2)^2+2.$$

□

32. Let $f(x) = x^2-3$ and $g(x) = 3x+1$. Find $(f\circ g)(x)$ and $(g\circ f)(x)$.

solution. $9x^2+6x-2$ and $3x^2-8$. □

33. Find all the roots of $2x^3+5x^2-28x-15$. *Hint: you should initially use the rational root theorem to find one root. This will allow you to reduce to a quadratic polynomial.*

solution. $3, -\dfrac{1}{2}, -5$. □

34. Find the midpoint of the line segment connecting $(3, 2)$ and $(7, 11)$.

solution. $(5, 13/2)$. □

35. $(x-7)^2 + (y+2)^2 - 16 = 9$. Find the center of this circle. Find the diameter.

solution. Center: $(7, -2)$. Diameter: 10. □

36. Write down the equation of a circle with midpoint $(5, 2)$ and radius 7.

solution. $(x-5)^2 + (x-2)^2 = 49$. □

37. Expand

$$\log\left(\frac{\sqrt{3x+2}}{x^2 y}\right).$$

solution. $\dfrac{1}{2}\log(3x+2) - 2\log x - \log y$. □

38. Combine $\ln y - 2\ln z + \dfrac{2}{3}\ln x$ into a single logarithm.

solution.

$$\ln\left(\frac{y x^{2/3}}{z^2}\right).$$

□

39. Solve

$$\ln(x-3) + \ln(x-6) = \ln 4.$$

Check for extraneous solutions.

solution. $x = 7$. □

40. Solve $e^{2x} - e^x = 56$. Check for extraneous solutions.

solution. $x = \ln 8$. □

41. Solve

$$e^{x^2 - x} = e^{x+3}.$$

solution. $x = 3$ or -1. □

42. Does the equation $y^2 - x = 2$ represent y as a function of x? Explain your answer.

No. Vertical line test.

43. The population P (in millions) of certain country is given by $P = 3e^{-t/60}$, where t is time in years.

(1) According to the model, is the population going up or down? Explain.
Answer: the population is going down because as time increases, the exponent becomes more negative.
(2) Find an expression for the time when the population will be 2 million.
Answer: $t = 60\ln(3/2)$.

44. Find the domain of $\ln(x^3 - 2)$.

solution. $x > 2^{1/3}$. □

45. If $e^{3x} = 5$, solve for x.

solution. $x = \dfrac{1}{3}\ln 5$. □

46. Condense the expression to a single logarithmic expression:

$$\log(x-3)+\log(2x+3)+\log(2-3x).$$

solution. $\log((x-3)(2x+3)(2-3x))$. □

47. Find the distance between $(2,3)$ and $(-3,2)$.

solution. $\sqrt{26}$ □

48. The cost per unit function for a manufacturing process is given by $f(x)=3x^2-48x+11$, where x represents the units produced. For what rate of production is the cost per unit minimized? What is the cost per unit at this rate?

Minimize the function by finding its vertex: $(8,-181)$. In other words, per unit cost is minimized at the rate of 8. The cost at this rate is -181. (This doesn't quite make sense, but you can still work through the math.)

49. Evaluate $f(x)=3x^4+2x^3-x^2+7x+6$ at $x=-2$ by synthetic division.

solution. The remainder after synthetic division with -2 is 20. □

50. Write $\dfrac{2+3i}{1-2i}$ in standard form. That is, write it in the form $a+bi$.

solution. $-\dfrac{4}{5}+\dfrac{7}{5}i$. □

51. Find the absolute value of $3-2i$.

solution. $\sqrt{13}$ □

52. Use the quadratic formula to find the roots of $3x^2+x+2$. Simplify as much as possible. Replace the sqaure root of negative one with i.

solution. $\dfrac{-1\pm i\sqrt{23}}{6}$. □

53. Suppose that \$500 is put in a CD with 7% yearly interest rate compounded three times per year.

(1) How much money is in the CD after 8 years?
Answer: $500(1+.07/3)^{24}$.

(2) How much time must pass before the balance grows to \$900? Give an exact expression by using logarithms.
Answer: $\dfrac{\log(9/5)}{3\log(1+.07/3)}$.

54. List all the possible rational roots of $7x^6-3x^4+12x^3-4x+11$.

solution. $\pm\dfrac{1}{7},\pm\dfrac{11}{7},\pm1,\pm11$. □

55. Graph the function 2^x. Label at least three points on your graph.

56. Rewrite $\log_2 3$ in terms of the natural logarithm and the logarithm with base 10.

solution. $\dfrac{\log 3}{\log 2},\dfrac{\ln 3}{\ln 2}$. □

57. Solve the equation $3\cdot 2^{3x-2}-4=3$ by using logarithms.

solution. $x=\dfrac{1}{3}\left(\dfrac{\ln(7/3)}{\ln 2}+2\right)$. □

58. Suppose that $a_4 = 7$ and $a_{10} = 25$ are terms of an arithmetic sequence. Find S_{50}, the sum of the first fifty terms.

solution. $25 \cdot 143$. □

59. Let $a_3 = 12$ and $a_5 = 3$ be terms of a geometric sequence. Find the infinite sum of the sequence.

solution. 96 □

60. Compute $\binom{10}{7}$.

solution. 120. □