#4: Expand $(2x+y)^4$

Section 9.5.

First, calculate binomial coefficients:

$$\binom{4}{0} = \frac{4!}{0!\,4!} = 1.$$

$$\binom{4}{1} = \frac{4!}{1!\,3!} = \frac{4\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{1\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 4$$

$$\binom{4}{2} = \frac{4!}{2!\,2!} = \frac{\cancel{4}\cdot 3\cdot 2\cdot 1}{\cancel{2\cdot 2}} = 6.$$

$$\binom{4}{3} = \frac{4!}{3!\,1!} = 4$$

$$\binom{4}{4} = \frac{4!}{4!\,0!} = 1.$$

$$(2x+y)^4 = \binom{4}{0}(2x)^4(y)^0 + \binom{4}{1}(2x)^3 y^1 + \binom{4}{2}(2x)^2 y^2$$

$$+ \binom{4}{3}(2x)^1 y^3 + \binom{4}{4}(2x)^0 y^4$$

$$= 1\cdot 2^4 x^4 + 4\cdot 2^3 x^3 y + 6\cdot 2^2 x^2 y^2$$

$$+ 4\cdot 2\cdot x\cdot y^3 + y^4$$

$= 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$

#5: Expand $(3x-2y)^3$

section 9.5

$$\binom{3}{0} = \frac{3!}{0!\,3!} = 1$$

$$\binom{3}{1} = \frac{3!}{1!\,2!} = \frac{3\cdot 2\cdot 1}{2} = 3$$

$$\binom{3}{2} = \frac{3!}{2!\,1!} = 3$$

$$\binom{3}{3} = \frac{3!}{3!\,0!} = 1.$$

$(3x-2y)^3 = (3x+(-2y))^3 =$

$\binom{3}{0}(3x)^3(-2y)^0 + \binom{3}{1}(3x)^2(-2y)^1 + \binom{3}{2}(3x)^1(-2y)^2$

$+ \binom{3}{3}(3x)^0(-2y)^3$

$= 1\cdot 3^3x^3 + 3\cdot 3^2x^2(-2y) + 3\cdot 3x\cdot 4y^2$

$+ 1\cdot(-2)^3y^3$

$= 27x^3 - 54x^2y + 36xy^2 - 8y^3.$

#6: $a_3 = 1/3 \quad a_7 = 625/3$ Find S_5.

Geometric sequence.

Section 9.3.

Formula:

$$a_n = a_1 r^{n-1}$$

$$a_3 = a_1 r^2 = 1/3 \qquad a_7 = a_1 r^6 = 625/3$$

$$\frac{a_7}{a_3} = \frac{\cancel{a_1} r^{\cancel{6}4}}{\cancel{a_1}\cancel{r^2}} = \frac{625/3}{1/3} = 625$$

$$r^4 = 625 \qquad r = 5.$$

$$a_3 = \frac{1}{3} = a_1 (5)^2$$

$$a_1 = \frac{1}{75}$$

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$S_5 = \frac{1}{75}\left(\frac{1-5^5}{1-5}\right) = \frac{1}{75}\left(\frac{-3124}{-4}\right) = \frac{781}{75}$$

8

Break even occurs when $f(x) = 0$.

$$102x - x^2 - 200 = 0$$

$$-(x-100)(x-2) = 0$$

$$x = 100, 2.$$

The firm makes a profit when $f(x) > 0$.

$$f(3) = 306 - 9 - 200 = 97 > 0.$$

Therefore, $f(x) > 0$ on the interval $(2, 100)$.

The firm makes a profit if it sells more than 2 or fewer than ~~100~~ 100 units.

#9: $f(x) = x - 95 - \dfrac{450}{x-10}$.

To find the roots of $f(x)$, we need to convert it to a single fraction:

$$f(x) = \frac{(x-95)(x-10) - 450}{x-10}$$

$$= \frac{x^2 - 95x - 10x + 950 - 450}{x-10}.$$

$$= \frac{x^2 - 105x + 500}{x-10}$$

The roots of f are the roots of its numerator.

$$x^2 - 105x + 500 = 0$$

$$(x-100)(x-5) = 0$$

$$x = 0, 5.$$

#19 Find a line perpindicular to $7x + 4y = 3$ with the same y-intercept.

Put the line in slope intercept form to find the y-intercept and slope:

$$4y = -7x + 3$$

$$y = \frac{-7}{4}x + \frac{3}{4} \qquad \text{y-intercept} = \frac{3}{4}$$

$$m = -\frac{7}{4} \qquad m_{perp} = \frac{-1}{m} = \frac{4}{7}$$

New line:

$$y = \frac{4}{7}x + \frac{3}{4}$$

#25 Multiply linear factors for each root:

$$\underset{3/2}{\underset{\uparrow}{(2x-3)}}\underset{-2}{\underset{\uparrow}{(x+2)}}\underset{1}{\underset{\uparrow}{(x-1)}}$$

Multiplying by a nonzero constant k does not change the roots of the above polynomial. Define

$$f(x) = k(2x-3)(x+2)(x-1)$$

y-intercept of f:

$$f(0) = k(-3)(2)(-1) = 6k = 5$$

$$k = 5/6.$$

#26 $\dfrac{2x^2+3x-2}{3x^2+x-6}$

The numerator and denominator have the same degree, so the horizontal asymptote is given by the ~~qoutient~~ quotient of the leading coefficients.

Horizontal asymptote at $y = 2/3$.

To find the vertical asymptotes, we must find the roots of the denominator. After attempts to factor fail, apply the quadratic formula:

$a=3 \quad b=1 \quad c=-6$

$$x = \frac{-1 \pm \sqrt{1-4(3)(-6)}}{6} = \frac{-1 \pm \sqrt{1+72}}{6}$$

$$= \frac{-1 \pm \sqrt{73}}{6}$$

#27:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

(1) $r = 5/100 = .05$ $\quad P = 500$

$t = 3 \quad n = 4$

$$A = 500(1 + .0125)^{4 \cdot 3}$$

$$= 500(1.0125)^{12}$$

(2) $A = 3000$

$$3000 = 500(1.0125)^{4t}$$

Simplify!

$$6 = (1.0125)^{4t}$$

Find t. Apply ln to each side:

$$\ln 6 = \ln\left((1.0125)^{4t}\right)$$

$$\ln 6 = 4t \ln(1.0125)$$

$$t = \frac{\ln 6}{4 \ln(1.0125)}$$

#31 Recall the standard form of a parabola:

$f(x) = a(x-h)^2 + k$ vertex $= (h, k)$.

$f(x) = a(x - 3/2)^2 + 2$.

Y-intercept:

$f(0) = a(3/2)^2 + 2 = 4$

Find a:

$a(3/2)^2 = 2$

$a\left(\frac{9}{4}\right) = 2$

$a = \frac{8}{9}$

#40:

This exponential equation is of quadratic type:

$$(e^x)^2 - e^x = 56$$

$$y = e^x$$

$$y^2 - y = 56$$

$$y^2 - y - 56 = 0$$

$$(y-8)(y+7) = 0$$

$$y = 8, -7.$$

-7 does not work because e^x must be positive.

$e^x = 8$ $\boxed{x = \ln 8}$

#41:

$$e^{x^2-x} = e^{x+3}$$

$$x^2 - x = x + 3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3, -1.$$

Plugging these numbers in to the equation, we find that both solutions are valid.

#43

$$2 = 3e^{-t/60}$$

$$\frac{2}{3} = e^{-t/60}$$

$$\ln\left(\frac{2}{3}\right) = \ln e^{-t/60}$$

$$\ln e^{-t/60} = \log_e e^{-t/60} = -t/60$$

$$\ln\left(\frac{2}{3}\right) = -t/60$$

$$t = -60 \ln\left(\frac{2}{3}\right).$$

$$= 60\left((-1)\ln\left(\frac{2}{3}\right)\right)$$

$$= 60\left(\ln\left(\left(\frac{2}{3}\right)^{-1}\right)\right)$$

$$= 60 \ln\left(\frac{3}{2}\right)$$

#44

The domain of the natural log is all positive numbers:

$$x^3 - 2 > 0$$

$$x^3 > 2$$

$$x > 2^{1/3}.$$

#45 $e^{3x} = 5$

$\ln e^{3x} = \ln 5$

$3x = \ln 5$

$x = \frac{1}{3} \ln 5$

Recall that

$\ln e^{x} = \log_e e^{x} = x.$

#48

We want to minimize $f(x)$. f is a parabola that opens upward because the coefficient of x^2 is positive. The minimum occurs at the vertex.

x-coordinate of vertex:

$$\frac{-b}{2a} = \frac{48}{2 \cdot 3} = 8.$$

Minimum cost is $f(8) = -181$.

(Note that in this case, it is probably easier to evaluate f by synthetic division than by plugging in.)

#57:

$$3 \cdot 2^{3x-2} - 4 = 3$$

$$3 \cdot 2^{3x-2} = 7$$

$$2^{3x-2} = \frac{7}{3}$$

$$\ln 2^{3x-2} = \ln(7/3)$$

$$(3x-2)\ln 2 = \ln(7/3)$$

$$3x-2 = \frac{\ln(7/3)}{\ln 2}$$

$$3x = \frac{\ln(7/3)}{\ln 2} + 2$$

$$x = \frac{1}{3}\left(\frac{\ln(7/3)}{\ln 2} + 2\right)$$

#66: compute $\binom{10}{7}$

$$\binom{10}{7} = \frac{10!}{7!\,3!} = \frac{10\cdot 9\cdot 8(7\cdot 6\cdots\cdots 2\cdot 1)}{7!\,3!}$$

$$= \frac{10\cdot 9\cdot 8\cdot \cancel{7!}}{\cancel{7!}\,3!} = \frac{10\cdot \overset{3}{\cancel{9}}\cdot 8}{\cancel{3}\cdot 2\cdot 1} = \frac{\overset{5}{\cancel{10}}\cdot 3\cdot 8}{\cancel{2}}$$

$$= 5\cdot 3\cdot 8 = 120.$$