

PROBLEMS:
DAY 10

- (1) Prove that the last digit of a triangular number cannot be 2, 4, 7 or 9.
(2) Show that if natural numbers a, b, c satisfy

$$a^2 + b^2 = c^2$$

then:

- (a) Either a or b is divisible by 3.
(b) Either a or b is divisible by 4.
(c) One of a, b and c is divisible by 5.
- (3) Prove that if $n > 4$ and n is not prime, then $(n - 1)!$ is divisible by n .
(4) Compute $x^{13} + \frac{1}{x^{13}}$ in terms of $a = x + \frac{1}{x}$.
(5) Show that $ax^3 + bx + c$ is an integer for each integer x if and only if $2a, a + b$ and c are integers.
(6) Show that if x, y, z and $\sqrt{x + y + z}$ are rational then so are \sqrt{x}, \sqrt{y} and \sqrt{z} .