1. Prove that

\[
\frac{|a - b|}{|1 - \bar{a}b|} < 1
\]

if $|a| < 1$, $|b| < 1$. 
2. Consider the collection $\mathcal{A}$ of all circles and straight lines in the complex plane $\mathbb{C}$. Show that elements of $\mathcal{A}$ are precisely solutions of equations:

$$az\bar{z} + bz + \bar{b}z + c = 0$$

where $a$ and $c$ are real numbers, and $b\bar{b} > ac$. Conclude the function $\varphi : \mathbb{C} \cup \infty \rightarrow \mathbb{C} \cup \infty$ defined as $\varphi(z) = 1/z$ maps circles into circles or straight lines, and maps straight lines into circles or straight lines.
3. Show that an analytic function \( f : U \to \mathbb{C} \), where \( U \) is an open, connected subset of \( \mathbb{C} \), cannot have a constant absolute value, without reducing to a constant.