History of Mathematics 3010

TEST 1

Your Name (Write CLEARLY in block letters):

INSTRUCTIONS

Work all 5 problems. SHOW YOUR WORK.

1. (10 points) Use marks "T" or "F".

   F (1) Pythagoras is known for discovery of regular polyhedra.
   F (2) Pythagoras learned number theory from Euclid.
   T (3) An ellipse can be obtained as a section of a cone by a plane.
   F (4) Using straight edge and a compass it is possible to double a cube.
   T (5) Archimedes was born in Sicily.
   T (6) Babylonians used number system based on base 60.
   T (7) Euclid knew that there are only five distinct regular polyhedra.
   F (8) Plimpton tablet 322 contains a proof of Pythagorean Theorem.
   F (9) Eudoxus noticed that each irrational number is a limit of rational numbers.
   T (10) A regular octahedron has six vertices.
2. (2 points) You are given two intervals: one of length 1 and one of length $x$. Explain how to construct an interval of length equal to $x^2$ having only a straight edge and a compass at your disposal.

Put $1$, $x$ on line 1

Draw line 2

Put $x$ on line 2

Draw int 1

Draw int 2

\[
\frac{x}{1} = \frac{y}{x} \quad \Rightarrow \quad x^2 = y
\]
3. (2 points) Using Cardano Rules, find one solution of the equation \( x^3 = 3x + 3 \).

Hint: Recall that if \( x^3 = px + q \), one can find a solution \( x \) which is of the form: \( x = u + v \), where \( u \) and \( v \) satisfy the equations \( u^3 + v^3 = q \) and \( 3uv = p \).

Use the hint:

\[
p = 3 \quad q = 3
\]

\[
3uv = 3 \quad \Rightarrow \quad uv = 1 \quad \Rightarrow \quad v = \frac{1}{u}
\]

Also,

\[
u^3 + v^3 = 3 \quad \Rightarrow \quad u^3 + \frac{1}{u^3} = 3 \quad \Rightarrow
\]

\[
(u^3)^2 + 1 = 3u^3 \quad \Rightarrow
\]

\[
(u^3)^2 - 3u^3 + 1 = 0
\]

This is a quadratic equation in \( u^3 \)

\[
\Rightarrow \quad u^3 = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}
\]

Take just one \( u \): say

\[
U = \frac{3 + \sqrt{5}}{2}
\]

Then

\[
V = \frac{1}{u} = \frac{2}{3 + \sqrt{5}}
\]

Hence

\[
X = \frac{3 + \sqrt{5}}{2} + \frac{2}{3 + \sqrt{5}}
\]
4. (2 points) Find all solutions of the equation \( x^3 = 3x + 2 \).

\[
x^3 - 3x - 2 = 0
\]

we guess \( x = -1 \) is a solution.

Check: \((-1)^3 - 3(-1) - 2 = -1 + 3 - 2 = 0 \checkmark\)

Divide \( x^3 - 3x - 2 \) by \( x - (-1) \)

\[
\begin{array}{c|ccc}
 & x^3 & +0x^2 & -3x & -2 \\
\hline
x+1 & x^3 & +x^2 & -3x & -2 \\
 & -x^3 & +x^2 & \\
\hline
 & -x^2 & -x & -2
\end{array}
\]

Hence \( x^3 - 3x - 2 = (x+1)(x^2 - x - 2) \)

Now, solve the equation \( x^2 - x - 2 = 0 \)

\[
X = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2}
\]

so \( x = 2, \text{ or } -1 \)

Ans: \( x = -1 \) (double root) \( \text{or } x = 2 \)
5. (2 points) Using Euclidean Algorithm, show that numbers 7201 and 7206 are relatively prime.

\[
\begin{array}{c|c|c}
7206 & 7201 & 5 \\
7201 & 5 \\
5 & 0 & \text{DONE}
\end{array}
\]

\(0\) is the greatest common divisor of 7201 and 7206.