1. Write an essay about development of Science, and in particular Mathematics in medieval Europe (roughly 476-1550). This essay should include the contributions of Alcuin, Gerald and Adelhard, Fibonacci, Regiomantanus and Luca Pacioli. You should not include Tartaglia, Cardano, Bombelli and Vieta, as their contributions are considered part of Renaissance.
2. Express \(y^2 - 4x - 12y + 28 = 0\) in a standard form. Identify the curve and graph it.

\[
y^2 - 4x - 12y + 28 = 0
\]

\[
y^2 - 12y + 36 - 12y + 28 = 36
\]

\[
(y - 6)^2 - 4x + 28 = 36
\]

\[
(y - 6)^2 = 4x + 8
\]

\(\star\) \[
x + 2 = \frac{1}{4}(y - 6)^2
\]

Let \(u = x + 2\)

\(v = y - 6\)

\(\Rightarrow \star \) become

\(\star \star \) \[
u = \frac{1}{4}v^2
\]

parabola
3. Express $4x^2 + y^2 + 40x - 2y + 97 = 0$ in a standard form. Identify the curve and graph it.

\[
4x^2 + y^2 + 40x - 2y + 97 = 0
\]
\[
4(x^2 + 10x) + (y^2 - 2y) = -97
\]
\[
4(x^2 + 10x + 25) + (y^2 - 2y + 1) = -97 + 100 + 1
\]
\[
4(x+5)^2 + (y-1)^2 = 9
\]
\[
\frac{(x+5)^2}{1^2} + \frac{(y-1)^2}{2^2} = 1
\]

ellipse centered at $(-5,1)$

with semi-axes 1 and 2
4. Express $x^2 + xy + y^2 = 6$ in a standard form. Identify the curve, and graph it showing the rotated axis.

$$x^2 + xy + y^2 = 6$$

Want to eliminate the cross term

$$x = u \cos \theta - v \sin \theta$$
$$y = u \sin \theta + v \cos \theta$$

$A = 1$, $B = 1$, $C = 1$

$$\cot (2\theta) = \frac{A-C}{B} = 0$$

$$\Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$$

$$\cos \theta = \frac{1}{\sqrt{2}}, \ \sin \theta = \frac{1}{\sqrt{2}}$$

$$x^2 + xy + y^2 = \frac{1}{2}(u-v)^2 + \frac{1}{2}(u-v)(u+v) + \frac{1}{2}(u+v)^2 = 6$$

After cancellation:

$$\frac{1}{2}(u^2 + v^2 - 2uv) + \frac{1}{2}(u^2 - v^2) + \frac{1}{2}(u^2 + v^2 + 2uv) = 6$$

$$\Rightarrow \frac{3}{2}u^2 + \frac{3}{2}v^2 = 6 \Rightarrow$$

$$\frac{u^2}{2^2} + \frac{v^2}{(2\sqrt{3})^2} = 1$$

an ellipse