Appendix

Mathematical Induction

Often in mathematics we are faced with the task of wanting to establish that a certain proposition \( P_n \) is true for every integer \( n \geq 1 \) (or perhaps every integer \( n \geq N \)). Here are three examples:

1. \( P_n: \quad 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n + 1)(2n + 1)}{6} \)
2. \( Q_n: \quad 2^n > n + 20 \)
3. \( R_n: \quad n^2 - n + 41 \) is prime

Proposition \( P_n \) is true for every positive integer, and \( Q_n \) is true for every integer greater than or equal to 5 (as we will show soon). The third proposition, \( R_n \), is interesting. Note that for \( n = 1, 2, 3, \ldots \), the values of \( n^2 - n + 41 \) are 41, 43, 47, 53, 61, \ldots (prime numbers so far). In fact, we will get a prime number for all \( n \)'s through 40; but at \( n = 41 \), the formula yields the composite number 1681 = (41)(41). Showing the truth of a proposition for 40 (or 40 million) individual cases may make a proposition plausible, but it most certainly does not prove it is true for all \( n \). The chasm between any finite number of cases and all cases is infinitely wide.

What is to be done? Is there a procedure for establishing that a proposition \( P_n \) is true for all \( n \)? An affirmative answer is provided by the Principle of Mathematical Induction.

Principle of Mathematical Induction

Let \( \{ P_n \} \) be a sequence of propositions (statements) satisfying these two conditions:
(i) \( P_1 \) is true (usually \( N \) will be 1).
(ii) The truth of \( P_i \) implies the truth of \( P_{i+1} \), \( i \geq N \).

Then, \( P_n \) is true for every integer \( n \geq N \).

We do not prove this principle; it is often taken as an axiom, and we hope it seems obvious. After all, if the first domino falls and if each domino knocks over the next one, then the whole row of dominos will fall. Our efforts will be directed toward illustrating how we use mathematical induction.

Prove that
\[ P_n: \quad 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n + 1)(2n + 1)}{6} \]
is true for all \( n \geq 1 \).

First, we note that
\[ P_1: \quad 1^2 = \frac{1(1 + 1)(2 + 1)}{6} \]
is a true statement.

Second, we demonstrate implication (ii). We begin by writing the statements \( P_i \) and \( P_{i+1} \):
\[ P_i: \quad 1^2 + 2^2 + \cdots + i^2 = \frac{(i + 1)(2i + 1)}{6} \]
\[ P_{i+1}: \quad 1^2 + 2^2 + \cdots + i^2 + (i + 1)^2 = \frac{(i + 1)(2i + 1)(2i + 3)}{6} \]
We must show that $P_j$ implies $P_{j+1}$, so we assume that $P_j$ is true. Then the left side of $P_{j+1}$ can be written as follows ($*$ indicates where $P_j$ is used):

$$\begin{align*}
[1^2 + 2^2 + \cdots + i^2] + (i + 1)^2 & \equiv \frac{(i + 1)(2i + 1)}{6} + (i + 1)^2 \\
& = (i + 1)2^2 + i + 6i + 6 \\
& = (i + 1)(i + 2)(2i + 3) \\
& \equiv 0 \iff i^2 + i + 6i + 6 \\
& \equiv i^2 + 4i^2 + 9 + 21.
\end{align*}$$

This chain of equalities leads to the statement $P_{j+1}$. Thus, the truth of $P_j$ does imply the truth of $P_{j+1}$. By the Principle of Mathematical Induction, $P_n$ is true for each positive integer $n$.

Prove that $P_n$: $2^n > n + 20$ is true for each integer $n \geq 5$.

First, we note that $P_5$: $2^5 > 5 + 20$ is true. Second, we suppose that $P_j$: $2^j > j + 20$ is true and attempt to deduce from this that $P_{j+1}$: $2^{j+1} > j + 21$ is true. But

$$2^{j+1} = 2 \cdot 2^j > 2(j + 20) = 2j + 40 > j + 21$$

Read from left to right, this is proposition $P_{j+1}$. Thus, $P_n$ is true for $n \geq 5$.

Prove that $P_n$: $x - y$ is a factor of $x^n - y^n$ is true for each integer $n \geq 1$.

Trivially, $x - y$ is a factor of $x - y$, so $P_1$ is true. Suppose that $x - y$ is a factor of $x^k - y^k$; that is,

$$x^k - y^k = Q(x, y)(x - y)$$

for some polynomial $Q(x, y)$. Then

$$x^{k+1} - y^{k+1} = x^{k+1} - x^k y + x^k y - y^{k+1}$$

$$= x^k(x - y) + y(x^k - y^k)$$

$$= x^k(x - y) + yQ(x, y)(x - y)$$

Thus, the truth of $P_j$ does imply the truth of $P_{j+1}$. We conclude by the Principle of Mathematical Induction that $P_n$ is true for all $n \geq 1$.

In Problems 1–8, use the Principle of Mathematical Induction to prove that the given proposition is true for each integer $n \geq 1$.

1. $1 + 2 + 3 + \cdots + n = \frac{n(n + 1)}{2}$
2. $1 + 3 + 5 + \cdots + (2n - 1) = n^2$
3. $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n + 1) = \frac{n(n + 1)(n + 2)}{3}$
4. $1^2 + 3^2 + 5^2 + \cdots + (2n - 1)^2 = \frac{n(2n - 1)(2n + 1)}{3}$
5. $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n + 1)}{2}\right)^2$
6. $1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n + 1)(6n^2 + 4n + 1)}{30}$
7. $n^5 - n$ is divisible by 6.
8. $n^3 + (n + 1)^3 + (n + 2)^3$ is divisible by 9.
In Problems 9–12, make a conjecture about the first integer $N$ for which the proposition is true for all $n \geq N$, and then prove the proposition for all $n \geq N$.

9. $3n + 25 < 3^n$
10. $n + 100 > \log_{10} n$
11. $n^2 \leq 2^n$
12. $|\sin x| \leq |\sin x|$ for all $x$

In Problems 13–20, indicate what conclusion about $P_n$ can be drawn from the given information.

13. $P_1$ is true, and $P_{i+1}$ true implies $P_{i+2}$ true.
14. $P_1$ and $P_2$ are true, and $P_{i+1}$ true implies $P_{i+2}$ true.
15. $P_0$ is true, and $P_{i+1}$ true implies $P_{i+2}$ true.
16. $P_0$ is true, and $P_i$ true implies both $P_{i+1}$ and $P_{i+2}$ true.
17. $P_i$ is true, and $P_{i+1}$ true implies $P_{i+2}$ true.
18. $P_i$ is true, and $P_{i+1}$ true for $j \leq i$ implies $P_{i+2}$ true.

In Problems 21–27, decide for what $n$’s the given proposition is true and then use mathematical induction (perhaps in one of the alternative forms that you may have discovered in Problems 13–20) to prove each of the following.

21. $x + y$ is a factor of $x^n + y^n$.
22. The sum of the measures of the interior angles of an $n$-sided convex (no holes or dents) polygon is $(n - 2)\pi$.
23. The number of diagonals of an $n$-sided convex polygon is $\frac{n(n - 3)}{2}$.
24. $\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n+2} \geq \frac{3}{5}$
25. $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\cdots\left(1 - \frac{1}{n}\right) = \frac{n+1}{2n}$
26. Let $f_0 = 0, f_1 = 1$, and $f_{n+2} = f_{n+1} + f_n$ for $n \geq 0$ (this is the Fibonacci sequence). Then $f_n = \frac{1}{\sqrt{5}}\left[\left(1 + \sqrt{5}\right)^n - \left(1 - \sqrt{5}\right)^n\right]$.
27. Let $a_0 = 0, a_1 = 1$, and $a_{n+2} = (a_{n+1} + a_n)/2$ for $n \geq 0$. Then $a_n = \frac{2}{\sqrt{5}}\left[1 - \left(-\frac{1}{2}\right)^n\right]$.
28. What is wrong with the following argument, which purports to show that all people in any set of $n$ people are the same age? The statement is certainly true for a set consisting of one person. Suppose that it is true for any set of $i$ people, and consider a set $W$ of $i + 1$ people. We may think of $W$ as the union of sets $X$ and $Y$, each consisting of $i$ people (draw a picture, for example, when $W$ has 6 people). By supposition, each of these sets consists of identically aged people. But $X$ and $Y$ overlap (in $X \cap Y$), and so all members of $W = X \cup Y$ also are the same age.

Main Limit Theorem

Proofs of Several Theorems

Let $n$ be a positive integer, $k$ be a constant, and $f$ and $g$ be functions that have limits at $c$. Then

1. $\lim_{x \to c} k = k$
2. $\lim_{x \to c} x = c$
3. $\lim_{x \to c} k f(x) = k \lim_{x \to c} f(x)$
4. $\lim_{x \to c} [f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$
5. $\lim_{x \to c} [f(x) - g(x)] = \lim_{x \to c} f(x) - \lim_{x \to c} g(x)$
6. $\lim_{x \to c} [f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$
7. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, provided $\lim_{x \to c} g(x) \neq 0$
8. $\lim_{x \to c} [f(x)]^n = [\lim_{x \to c} f(x)]^n$
9. $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$, provided $\lim_{x \to c} f(x) > 0$ when $n$ is even.

Proof We proved parts 1 through 5 near the end of Section 1.3, so we should start with part 6. However, we choose first to prove a special case of part 8:

$\lim_{x \to c} [g(x)]^2 = [\lim_{x \to c} g(x)]^2$

To see this, recall that we have proved that $\lim_{x \to c} x^2 = c^2$ (Example 7 of Section 1.2), and so $f(x) = x^2$ is continuous everywhere. Thus, by the Composite Limit Theorem (Theorem 1.6E).
\[
\lim_{x \to a} (f(x))^2 = \lim_{x \to a} f(x)^2 = \left( \lim_{x \to a} f(x) \right)^2
\]

Next, write
\[
f(x)g(x) = \frac{1}{4} \left( [f(x) + g(x)]^2 - [f(x) - g(x)]^2 \right)
\]
and apply parts 3, 4, and 5, plus what we have just proved. Part 6 is proved.

To prove part 7, apply the Composite Limit Theorem with \( f(x) = \frac{1}{x} \) and use Example 6 of Section 1.2. Then
\[
\lim_{x \to a} \frac{1}{g(x)} = \lim_{x \to a} f(g(x)) - \lim_{x \to a} f(g(x)) = \frac{1}{\lim_{x \to a} g(x)}
\]
Finally, by part 6,
\[
\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \left[ f(x) - \frac{1}{g(x)} \right] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} \frac{1}{g(x)}
\]
from which the result follows.

Part 8 follows from repeated use of part 6 (technically, by mathematical induction).

We prove part 9 only for square roots. Let \( f(x) = \sqrt{x} \), which is continuous for positive numbers by Example 5 of Section 1.2. By the Composite Limit Theorem,
\[
\lim_{x \to a} \sqrt{g(x)} = \lim_{x \to a} f(g(x)) = \sqrt{\lim_{x \to a} g(x)}
\]
which is equivalent to the desired result. ⬜

**Chain Rule**

If \( g \) is differentiable at \( a \) and \( f \) is differentiable at \( g(a) \), then \( f \circ g \) is differentiable at \( a \) and
\[
(f \circ g)'(a) = f'(g(a))g'(a)
\]

**Proof** We offer a proof that generalizes easily to higher dimensions (see Section 12.6). By hypothesis, \( f \) is differentiable at \( b = g(a) \), that is, there is a number \( f'(b) \) such that
\[
(1) \quad \lim_{\Delta u \to 0} \frac{f(b + \Delta u) - f(b)}{\Delta u} = f'(b)
\]
Define a function \( \varepsilon \) depending on \( \Delta u \) by
\[
\varepsilon(\Delta u) = \frac{f(b + \Delta u) - f(b)}{\Delta u} - f'(b)
\]
and multiply both sides by \( \Delta u \) to obtain
\[
(2) \quad f(b + \Delta u) - f(b) = f'(b) \Delta u + \Delta u \varepsilon(\Delta u)
\]
The existence of the limit in (1) is equivalent to \( \varepsilon(\Delta u) \to 0 \) as \( \Delta u \to 0 \) in (2). If, in (2), we replace \( \Delta u \) by \( g(a + \Delta x) - g(a) \) and \( b \) by \( g(a) \), we get
\[
f(g(a + \Delta x)) - f(g(a)) = f'(g(a))[g(a + \Delta x) - g(a)] + [g(a + \Delta x) - g(a)] \varepsilon(\Delta u)
\]
or, upon dividing both sides by \( \Delta x \),
\[
(3) \quad \frac{f(g(a + \Delta x)) - f(g(a))}{\Delta x} = f'(g(a)) \frac{g(a + \Delta x) - g(a)}{\Delta x} + \frac{g(a + \Delta x) - g(a)}{\Delta x} \varepsilon(\Delta u)
\]
In (3), let \( \Delta x \to 0 \). Since \( g \) is differentiable at \( a \), it is continuous there, so \( \Delta x \to 0 \); this, in turn, makes \( e(\Delta x) \to 0 \). We conclude that

\[
\lim_{\Delta x \to 0} \frac{f(g(a + \Delta x)) - f(g(a))}{\Delta x} = f'(g(a)) \lim_{\Delta x \to 0} \frac{g(a + \Delta x) - g(a)}{\Delta x} + 0
\]

That is, \( f \circ g \) is differentiable at \( a \) and

\[
(f \circ g)'(a) = f'(g(a))g'(a)
\]

**Power Rule**

If \( r \) is rational, then \( x^r \) is differentiable at any \( x \) that is in an open interval on which \( x^{r-1} \) is real and

\[
D_x(x^r) = rx^{r-1}
\]

**Proof** Consider first the case where \( r = \frac{1}{q} \), \( q \) a positive integer. Recall that \( a^r - b^r \) factors as

\[
a^r - b^r = (a - b)(a^{r-1} + ar^{-2} b + \cdots + ab^{r-2} + b^{r-1})
\]

so

\[
a^r - b^r = \frac{1}{(a^{r-1} + ar^{-2} b + \cdots + ab^{r-2} + b^{r-1})}
\]

Thus, if \( f(t) = t^{1/q} \),

\[
f'(x) = \lim_{t \to x} \frac{t^{1/q} - x^{1/q}}{t - x} = \lim_{t \to x} \frac{t^{1/q} - x^{1/q}}{(x^{1/q})^q - (t^{1/q})^q} = \frac{1}{q(x^{(q-1)/q} + t^{-(q-1)/q} + \cdots + x^{(q-1)/q})} = \frac{1}{qx^{(q-1)/q}}
\]

Now, by the Chain Rule, and with \( p \) an integer,

\[
D_x(x^{p/q}) = D_x[(x^{1/q})^p] = px^{p/q - 1}\quad D_x(x^{1/q}) = px^{p/q - 1} = \frac{p}{q} x^{p/q - 1}
\]

**Vector Limits**

Let \( F(t) = f(t)\mathbf{i} + g(t)\mathbf{j} \). Then \( F \) has a limit at \( c \) if and only if \( f \) and \( g \) have limits at \( c \). In that case,

\[
\lim_{t \to c} F(t) = \left[ \lim_{t \to c} f(t) \right] \mathbf{i} + \left[ \lim_{t \to c} g(t) \right] \mathbf{j}
\]

**Proof** First, note that for any vector \( \mathbf{u} = u_1 \mathbf{i} + u_2 \mathbf{j} \),

\[
|\mathbf{u}| = |\mathbf{u}| = |u_1| + |u_2|
\]

This fact is readily seen from Figure 1.

Now suppose that \( \lim_{t \to c} F(t) = L = a \mathbf{i} + b \mathbf{j} \). This means that for any \( \varepsilon > 0 \) there is a corresponding \( \delta > 0 \) such that

\[
0 < |t - c| < \delta \implies |F(t) - L| < \varepsilon
\]
But, by the left part of the boxed inequality,
\[ |f(t) - a| \leq |F(t) - L| \]
and so
\[ 0 < |r - c| < \delta \Rightarrow |f(r) - a| < \varepsilon \]
This shows that \( \lim_{t \to c} f(t) = a \). A similar argument establishes that \( \lim_{t \to c} g(t) = b \).
The first half of our theorem is complete.

Conversely, suppose that
\[ \lim_{t \to c} f(t) = a \quad \text{and} \quad \lim_{t \to c} g(t) = b \]
and let \( L = a\mathbf{i} + b\mathbf{j} \). For any given \( \varepsilon > 0 \), there is a corresponding \( \delta > 0 \) such that
\[ 0 < |r - c| < \delta \] implies that both
\[ |f(r) - a| < \frac{\varepsilon}{2} \quad \text{and} \quad |g(r) - b| < \frac{\varepsilon}{2} \]
Hence, by the right part of the boxed inequality,
\[ 0 < |r - c| < \delta \Rightarrow |F(r) - L| \leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon \]
Thus,
\[ \lim_{t \to c} F(t) = L = a\mathbf{i} + b\mathbf{j} = \lim_{t \to c} f(t)\mathbf{i} + \lim_{t \to c} g(t)\mathbf{j} \]
\[ \blacksquare \]
Answers to Odd-Numbered Problems

1. 16. 3. -148 5. \( \frac{20}{3} \) 7. \( \frac{4}{3} \) 9. \( \frac{8}{5} \) 11. \( \frac{3}{9} \)

13. 15. \( \frac{2}{3} \) 17. \( 3x^2 - x - 4 \) 19. \( 6x^2 - 13x - 9 \)

21. \( 3x^3 + 6x^2 + 9x = 3 \) 23. \( x + 2, x \neq 2 \)

25. \( x - 7, x \neq -3 \) 27. \( \frac{2(3x + 10)}{x(x + 2)} \)

31. \( 0.08333... \) 33. \( 0.142857... \) 35. \( 3.6666... \)

37. \( \frac{1}{3} \) 39. \( \frac{2}{3} \) 41. \( \frac{5}{3} \)

43. Those rational numbers that can be expressed by a terminating decimal followed by zeros

47. Those rational numbers that can be expressed by a terminating decimal followed by zeros

51. 20.39230485 53. 0.00028307388

55. 0.000691744752 59. 132,700,874 ft

61. 651.441 board ft

65. (a) \( \frac{1}{2} \) (b) \( \frac{2}{3} \) (c) \( \frac{3}{4} \) (d) \( \frac{4}{5} \) (e) \( \frac{5}{6} \)

67. (a) True; (b) False; (c) False; (d) True; (e) True

71. (a) True; (b) False; (c) False; (d) True; (e) True

75. (a) 3-3-3-3-3 or 3\(^3\) (b) 2-2-3-1 or 2\(^2\) - 3 - 1

81. (a) Rational; (b) Rational; (c) Rational; (d) Irrational

1. (a) \( -2, -\infty \) (b) \( -3, -1 \) (c) \( -2, -1 \) (d) \( -1, 0 \) (e) \( 0, 1 \)

5. \( -\infty, -3 \) (6) \( 3, \infty \) (7) \( 7, \infty \) (8) \( 1, \infty \)

9. \( \sqrt{170} \)
A-8 Answers to Odd-Numbered Problems

13. \((x - 2)^2 + (y + 1)^2 = 25\)
15. \((x - 2)^2 + (y - 5)^2 = 5\)
17. Center = \((-1, 3)\); radius = \(\sqrt{10}\)
19. Center = \((6, 0)\); radius = 1
21. Center = \((-2, -\frac{1}{2})\); radius = \(\frac{\sqrt{13}}{4}\)
23. 1, 25, 7, \(-\frac{7}{12}\), 29, \(y = -x + 4; x + y = 4 = 0\)
31. \(y = 2x + 3; 2x - y + 3 = 0\)
33. \(y = \frac{1}{2}x - 2; 5x - 2y = 4 = 0\)
35. Slope = \(-\frac{1}{2}; \) y-intercept = \(\frac{1}{2}\)
37. Slope = \(-\frac{1}{2}; \) y-intercept = \(\frac{1}{2}\)
39. (a) \(y = 3x - 9\); (b) \(y = -\frac{1}{3}x - \frac{1}{2}\); (c) \(y = -\frac{1}{3}x - 1\);
41. \(y = \frac{3}{2}x + 2\) 43. It lies above the line.
45. \((-1, 2); y = \frac{3}{2}x + \frac{3}{2}\) 47. \((3, 1); y = -\frac{1}{3}x + 5\)
49. Inscribed: \((x - 4)^2 + (y - 1)^2 = 4; \) circumscribed: \((x - 4)^2 + (y - 1)^2 = 8\)
55. \(d = 2\sqrt{3} + 4\) 64. \(18 + 2\sqrt{17} + 4\pi = 38.8\)
63. \(\frac{2}{3}\; 65.\; \frac{18}{13}\; 67.\; \sqrt{\frac{5}{3}}\; 69.\; y = \frac{2}{3}x + \frac{3}{2}\)
73. \(x + \sqrt{\frac{5}{3}} = 12\) and \(x - \sqrt{\frac{5}{3}} = 12\)
77. \(\frac{1}{3}\)

1. 3
5. 7
9. 11
13.
15.
5. (a) Undefined; (b) $2.658$; (c) $0.841$

7. (a) Not a function; (b) $f(x) = \frac{1}{x + 1}$; (c) $f(x) = \frac{1}{3} - x$

9. $4a + 2b$

11. $-x^4 - 4x + 6x - 2a + 4$

13. (a) $\{x \in \text{reals: } x = \frac{1}{2}\}$; (b) $\emptyset$; (c) $\{x \in \text{reals: } |x| \leq 3\}$; (d) $\{y \in \text{reals: } |y| \leq 5\}$

15. Even

17. Neither

19. Neither

21. Odd

23. Neither

25. Even

27. Neither

29. Neither

31. $T(x) = 5000 + 805x; \{x \in \text{integers: } 0 \leq x \leq 100\}$

$u(x) = \frac{200}{x} + 805; \{x \in \text{integers: } 0 < x \leq 100\}$

33. $E(x) = x - x^2$

35. $E(x) = \sqrt{y^2 - x^2}$

37. (a) $E(x) = 24 + 0.40x$; (b) $240$ miles

39. $A(d) = \frac{2d - \pi d^2}{4}$

41. (a) $b(0) = 0$; (b) $b(1) = \frac{1}{2}$

43. $r = 3.7$

45. (a) $f(1.38) \approx 0.2994, f(4.12) = 3.6852$

47. (a) $\{y \in \text{reals: } -22 \leq y \leq 13\}$

(b) $[-1.1, 1.7] \cup [4.3, 5]$

49. (a) $x$-intercept $\frac{4}{3}$, $y$-intercept $\frac{2}{3}$

(b) all reals

(c) $x = -3, x = 2$

(d) $y = 0$
15.  

17.  

19.  

21.  

23. (a) Even; (b) Odd; (c) Even; (d) Even; (e) Odd

25. No, in both cases. (Consider \( f(x) = x^2 + x \) and \( f(x) = x^3 + 1 \).)

27. (a) \( P = \sqrt{x + \sqrt{x + 27}} \); (b) \( P = 7 \)

29. \( D(r) = \frac{4000}{\sqrt{250000r^2 - 180000r + 90000}} \) if \( 0 \leq r \leq 1 \)

31. (a) \( \frac{1}{1 - r} \); (b) \( x \); (c) \( 1 - x \)

37.  

39.  

41. (a)  

(b)  

1. (a) \( \frac{\pi}{2} \); (b) \( \frac{\pi}{3} \); (c) \( -\frac{\pi}{3} \); (d) \( \frac{3\pi}{4} \); (e) \( -\frac{\pi}{4} \); (f) \( \frac{\pi}{4} \)

3. (a) 0.5812; (b) 0.8029; (c) -1.1624; (d) 4.1907; (e) -6.4403; (f) 0.9520

5. (a) 68.37; (b) 0.8545; (c) 0.4855; (d) -0.3532; (e) 46.997; (f) 0.0789

7. (a) \( \frac{\sqrt{3}}{3} \); (b) -1; (c) \( -\sqrt{2} \); (d) 1; (e) 1; (f) -1

9. (a) 46.097; (b) -0.8845; (c) 0.4855; (d) -0.3532; (e) -6.4403; (f) 0.9520

15. (a)  

(b)  

17. Period = \( \pi \); Amplitude = 2

19. Period = \( \frac{\pi}{2} \); shift 2 units up
21. Period = \( \pi \); amplitude = 7; shift: 21 units up, \( \frac{1}{2} \) units left

23. Period = \( \frac{\pi}{2} \); shift: \( \frac{1}{2} \) units right

25. (a) Even; (b) Even; (c) Odd; (d) Even; (e) Even; (f) Odd

27. \( \frac{1}{3} \); (b) \( \frac{1}{3} \)

31. \( \frac{2}{\sqrt{2}} \)

35. 336 rev/min

37. 28 rev/sec

39. (a) \( f \); (b) \( \sqrt{2} \)

41. (a) 6.1419; (b) 1.8925; (c) 1.7127

43. 25 cm

45. \( r^2 \sin x \cos x + \sin^2 x = \frac{1}{4} \)

47. 67.5°F

49. As \( t \) increases, the point on the rim of the wheel will move around the circle of radius 2.

(a) \( r(2) = 1.902; y(2) = 0.618; x(6) = -1.176; y(6) = -1.618; x(0) = 0; y(0) = 2 \)

(b) \( x(t) = -2 \sin \left( \frac{\pi}{2} t \right); y(t) = 2 \cos \left( \frac{\pi}{2} t \right) \)

(c) The point is at \( (2, 0) \) when \( \frac{\pi}{2} t = \frac{\pi}{2} \); that is, when \( t = 1 \).

51. (c) \( A_1 \sin(x + \phi_1) + A_2 \sin(x + \phi_2) = \left( A_1 \cos \phi_1 + A_2 \cos \phi_2 \right) x + \left( A_1 \sin \phi_1 + A_2 \sin \phi_2 \right) \sin x \cos \phi_1 + \left( A_1 \sin \phi_1 + A_2 \sin \phi_2 \right) \cos x \cos \phi_2 \)

53. (a) 

(c) 


41. True 43. True 45. False 47. True 49. True


61. True 63. False
47. \( V(x) = x(32 - 2x)(24 - 2x), \{ x \text{ real} \mid 0 \leq x \leq 12 \} \)

49. (a) 

![Graph](image)

(b) 

![Graph](image)

(c) 

![Graph](image)

51. \( f(x) = \sqrt{x}, g(x) = 1 + x, h(x) = x^2, k(x) = \sin x \)

53. (a) -0.6; (b) -0.6; (c) -0.96; (d) -1.333

(e) 0.8; (f) -0.8

55. 6

57. 3

37. Does not exist

39. (a) Does not exist; (b) 0

41. \( a = -1, 0, 1 \)

43. (a) Does not exist; (b) -1; (c) -3; (d) Does not exist

45. (a) 1; (b) 0; (c) -1; (d) -1

47. Does not exist

49. 0

51. \( \frac{1}{2} \)

53. Does not exist

55. 6

57. -3

1. \( 0 < \lvert x - a \rvert < \delta \Rightarrow \lvert f(x) - L \rvert < \epsilon \)

3. \( 0 < \lvert x - d \rvert < \delta \Rightarrow \lvert h(x) - P \rvert < \epsilon \)

5. \( 0 < c - x < \delta \Rightarrow \lvert f(x) - L \rvert < \epsilon \)

7. 0.001

9. 0.0019

31. (b), (c)

33. (a) \( \frac{x^2 - x^2 - 2x - 4}{x^3 - 4x^2 + x^3 + x} \)

(b) No.

(e) \( x^3 \)

1. 3

3. -3

5. -5

7. 2

9. -1

11. 2

13. 0

15. -4

17. -\frac{4}{25}

19. \frac{1}{2}

21. \frac{x + 2}{5}

23. -1

25. \( \sqrt{10} \)

27. -6

29. 6

31. 12

33. -\frac{1}{4}

41. 0

43. 0

45. \( \frac{1}{2} \)

47. -1

51. (a) 1; (b) 0

1. 1

3. 1

5. 3

7. 3

9. \frac{1}{27}

11. 0

13. 7

15. 0

17. 0

19. 2
1. Continuous
3. Not continuous; \( \lim_{x \to 3} \frac{3}{x-3} \) and \( h(3) \) do not exist.
5. Not continuous; \( \lim_{x \to 3} \frac{3}{x-3} \) and \( h(3) \) do not exist.
7. Continuous
9. Not continuous; \( h(3) \) does not exist.
11. Continuous
13. Continuous
15. Continuous
17. \( (-\infty, -5), (-5, 4), (4, 6), (6, 8), (8, \infty) \)
19. Define \( f(3) = -12 \).
21. Define \( H(1) = \frac{1}{2} \).
23. Define \( F(-1) = -\sin 2 \).
25. \( 3, \pi \)
27. Every \( \theta = n + \frac{\pi}{2} \) where \( n \) is any integer.
29. \(-1\)
31. \( (-\infty, -2) \cup [2, \infty) \)
33. Every \( t = n + \frac{1}{2} \) where \( n \) is any integer.
37. Define \( f(x) = -12 \).
41. Define \( g(x) = \frac{1}{2} \).
43. Define \( h(x) = -\sin 2 \).
45. Define \( k(-1) = -\sin 2 \).
47. Define \( m(x) = \frac{1}{2} \).
49. Define \( n(x) = -12 \).
51. The function is continuous on the intervals \( (0, 0.1], (0.1, 0.25], (0.25, 0.375], (0.375, 0.5], ... \)
53. \( 0 \)
55. \( 0 \)
57. \( 0 \)
59. \( -3 \)
61. \( 1 \)
63. \( 0 \)
65. \(-1\)
67. \( -\infty \)
69. \( e \)
71. \( 1 \)

43. Horizontal asymptote \( y = 0 \)
Vertical asymptote \( x = -1 \)

45. Horizontal asymptote \( y = 2 \)
Vertical asymptote \( x = 3 \)

47. Horizontal asymptote \( y = 0 \)
No vertical asymptotes

49. The oblique asymptote is \( y = 2x + 3 \).

51. (a) We say that \( \lim_{x \to \infty} f(x) = -\infty \) if for each negative number \( M \) there corresponds a \( \delta > 0 \) such that \( 0 < x - c < \delta \Rightarrow f(x) < M \).
(b) We say that \( \lim_{x \to \infty} f(x) = \infty \) if for each positive number \( M \) there corresponds a \( \delta > 0 \) such that \( 0 < c - x < \delta \Rightarrow f(x) > M \).

55. (a) Does not exist.
(b) 0
(c) 1
(d) \( \infty \)
(e) \( 0 \)
(f) \( \infty \)
(g) Does not exist.
(h) 0

57. \( 0 \)
61. \( 1 \)
63. \( 0 \)
65. \(-1\)
67. \( -\infty \)
69. \( e \)
71. \( 1 \)

1. Continuous
3. Not continuous; \( \lim_{x \to 3} \frac{3}{x-3} \) and \( h(3) \) do not exist.
5. Not continuous; \( \lim_{x \to 3} \frac{3}{x-3} \) and \( h(3) \) do not exist.
7. Continuous
9. Not continuous; \( h(3) \) does not exist.
11. Continuous
13. Continuous
15. Continuous
17. \( (-\infty, -5), (-5, 4), (4, 6), (6, 8), (8, \infty) \)
19. Define \( f(3) = -12 \).
21. Define \( H(1) = \frac{1}{2} \).
23. Define \( F(-1) = -\sin 2 \).
25. \( 3, \pi \)
27. Every \( \theta = n + \frac{\pi}{2} \) where \( n \) is any integer.
29. \(-1\)
31. \( (-\infty, -2) \cup [2, \infty) \)
33. Every \( t = n + \frac{1}{2} \) where \( n \) is any integer.
37. Define \( f(x) = -12 \).
41. Define \( g(x) = \frac{1}{2} \).
43. Define \( h(x) = -\sin 2 \).
45. Define \( k(-1) = -\sin 2 \).
47. Define \( m(x) = \frac{1}{2} \).
49. Define \( n(x) = -12 \).
51. The function is continuous on the intervals \( (0, 0.1], (0.1, 0.25], (0.25, 0.375], (0.375, 0.5], ... \)
53. \( 0 \)
55. \( 0 \)
57. \( 0 \)
59. \( -3 \)
61. \( 1 \)
63. \( 0 \)
65. \(-1\)
67. \( -\infty \)
69. \( e \)
71. \( 1 \)

43. Continuous
45. Discontinuous: removable, redefine \( g(0) = 1 \)
47. Discontinuous: nonremovable.
49. Horizontal asymptote \( y = 0 \)
No vertical asymptotes
51. The function is continuous on the intervals \( (0, 0.1], (0.1, 0.25], (0.25, 0.375], (0.375, 0.5], ... \)
55. The interval \([0.6, 0.7]\) contains the solution.
65. Yes, \( g \) is continuous.
71. (a) Domain \( \left[ \frac{3}{4}, \frac{3}{4} \right] \)
(b) Discontinuous at \( x = 0 \)
(c) \( \frac{3}{4} \)
A-14 Answers to Odd-Numbered Problems


3. 0  3. 2  5. 2  7. 2  9. 4  11. -1  13. -1
15. 3  17. 1  19. ∞  21. ∞

25. (a) x = -1.1  (b) f(-1) = -1
27. (a) 14  (b) -12  (c) -2  (d) -2  (e) 5  (f) 0
29. a = 2, b = -1
31. Vertical none, Horizontal y = 0
33. Vertical: x = ±π/2, ±3π/2, ±3π/2, ..., Horizontal none

1. (a) 4  (b) 4.41  (c) 0.41  (d) 4.1  (e) a² + 2ah + h²
(f) 2ah + h²  (g) 2a + h  (h) 2a
3. (a) √2 = 1.41  (b) √2.1 = 1.45  (c) 0.035  (d) 0.35
(e) √a + h  (f) √a + h - √a  (g) (√a + h - √a)/h
(h) 1/2
5. (a) a² + 3a²b  (b) a² + 4a²b  (c) a² + 5a²b
7. sin(x + h) = sin x cos h + cos x sin h

11. (a) North plane has traveled 600 miles. East plane has traveled 400 miles.  (b) 721 miles
33. 2.818

1. 4  3. -2  5. 2  7. 2  9. 4
11. x = ±π/2, ±3π/2, ±3π/2, ..., Horizontal none

39.
Answers to Odd-Numbered Problems A-15

45. 1.5 47. -0.1667 49. 0.0081 51. 2x
53. -1/(x + 1)^2 55. 2/(x + 1)^2 57. 1, 3, 5
61. (a) 0.5; (b) 0.5; (c) 3, 5; (d) 1, 3, 5; (f) 0; (g) -0.7, 1.5, 5.7
63. r - 1 - f, g = f is solid; g’ is long-dashed
65. f is short-dashed; g = f’ is solid; g’ is long-dashed
67. m = 4, b = -4
69. (a) m; (b) -m
71. (a) (0, 4); (b) [0, 4]; (c) f(x) decreases as x increases when f’(x) < 0.

Problems Set 2.5

1. 4x 3. -x 5. -4x^3 7. -\pi/3 9. -500/x^3
11. 2x + 2 13. 4x^3 + 3x^2 + 2x + 1
15. 7x^6 - 10x^4 + 10x^3 - 9x^2 + 4x^5
19. 2/x^2 + 2/x 21. -1/2x^2 + 2 23. 3x^2 + 1
25. 8x + 4 27. 5x^4 + 6x^2 + 2x
29. 5x^4 + 42x^2 + 2x - 31 31. 60x^3 - 30x^2 - 32x + 14
33. \frac{1}{10} 6x 35. \frac{1}{10} \left(\frac{20x - 3}{3x + 5}\right)^2
37. \frac{1}{10} \left(\frac{8x - 3}{3x + 5}\right)^2
39. \frac{4x^2 + 4x - 5}{\left(\frac{20x + 3}{3x + 5}\right)^2} 41. \frac{4x^2 + 4x - 5}{\left(\frac{20x + 3}{3x + 5}\right)^2}
43. \frac{x^2 - 1}{\left(\frac{20x + 3}{3x + 5}\right)^2}

45. (a) 2; (b) 4; (c) -2/x 47. y = 1 51. 0.0 (0) and (\sqrt{3}, \sqrt{3})
53. (2.817, 0.563) and (-2.817, -0.563)
55. (a) -24 ft/sec (b) 1.25 s
57. y = 2x + 1, y = -2x + 9 59. 3\sqrt{5}
61. 881 cm^3 per week

1. 2 \cos x - 3 \sin x 3. 0 5. \sec x \tan x 7. \sec^2 x
9. \sec^3 x 11. \cos x - \sin x 13. \frac{x^2 \cos x - \sin x}{x^2}
15. -x^3 \sin x + 2x \cos x 17. 2 \tan x \sec^2 x
19. y = 0.5, y = -0.8415(x - 1) 21. -2 \sin x + 2 \cos x
23. 3\sqrt{5} ft/sec 25. y = x
27. x = 2/3 + k\pi where k is an integer.

33. (a) (b) 6.5; (c) f(x) = x \sin x with a = 0 and b = \pi is a counterexample; (d) 24.93

1. 15(1 + x)^4 3. -6(3 - 2x)^6
5. 11(3x^2 - 4x + 3)(x^2 - 2x + 3x + 1)^10
7. -\frac{x}{(x + 3)^2} 9. (2x + 1) \cos(x^2 + x)
11. -3 \sin \pi \cos^2 x 13. -6(x + 1)^2
15. -3x^2 + 12x (x + 2)^2 \sin \left(\frac{3x^2}{x + 2}\right)
17. 2(3x - 2)(3 - x^2)(9 + 4x - 9x^2)
19. \frac{(x + 1)(3x - 11)}{(3x - 4)} 21. 4x(x^2 + 4)
23. \frac{5(3x - 2)^2}{(r + 5)^2} 25. \frac{(6x + 47)(3x - 2)^2}{(r + 5)^2}
27. 3 \sin^2 x(x \cos x + 2 \cos x + 2) \cos^2 x
29. 2x

31. 1.4183 33. 4(2x + 3) \sin \frac{\pi}{2} \cos^2 x 35. -3 \sin \theta \sin^2 \theta \cos \theta \cos \theta
37. -80 \cos^3 \sin^2 \theta \cos \theta \sin \theta
39. -2 \cos \cos (\sin 2x) \sin (\sin 2x) \cos (2x)
41. 2 43. 1 45. -1 47. 2F'/(2x)
49. -2F/(F(t))F'(t) 51. 0(1 + F(2x))/F'(2x)
53. -\sin \theta \cos \theta \cos (\sin \theta) \cos \theta
55. 2F' \sin F(x) \cos F(x) + F'(x) \sin^2 F(x)
57. -2 \sin x 61. -1 63. x = w/4 + kw, k = 0, 1, 2, ...;
65. y = \frac{1}{2}x + \frac{3}{2} 67. x = 3/2
69. (a) (10 cos 8\pi x, 10 sin 8\pi x); (b) 80\pi cm/s
71. (a) (cos 2\pi x, sin 2\pi x); (b) sin 2\pi x + \sqrt{25 - \cos^2 2\pi x};
60. 2\pi \cos 2\pi x \left(1 - \frac{\sin 2\pi x}{\sqrt{25 - \cos^2 2\pi x}}\right)
73. 0.38 in/min 75. x_0 = w/3; \theta = 1.25 rad.
79. cot \theta \tan \theta \sec \theta \csc \theta
81. 16

1. 6 3. 162 5. -343 \cos(7x) 7. -\frac{6}{(x - 1)^2}
9. 2 11. 1 13. 2\pi^2 15. -900
A-16 Answers to Odd-Numbered Problems

19. (a) 0; (b) 0; (c) 0
21. \( f'(x) = 3x^2 - 18x + 24; f'(x) = 6x - 18; \)
   \( (-\infty, 2) \cup (4, \infty); \) (c) (2, 4); (d) \(-\infty, 3); \)
23. (a) \( f(x) = 12 - 4x; \) \( a(x) = -4 \)
   \( (0, \infty); \) (d) All \( x; \) (e) \( \phi \)
25. (a) \( v(t) = 3t^2 - 18t + 24; \) \( a(t) = -4 \)
   \( (-\infty, 3) \cup (4, \infty); \) (c) (2, 4); (d) \(-\infty, 3); \)
27. (a) \( f(t) = 2t - 16; \) \( f(t) = 2 \)
   \( (0, 2); \) (b) \( (2, \infty); \)

29. \( r(4) = 11; \) \( r(4) = -16 \)
31. (a) \( \frac{1}{2}; \) (b) \( \frac{3}{2}; \) (c) \( 0, \frac{1}{2} \)
33. (a) 48 ft/s; (b) \( \frac{1}{2}; \) (c) 292 ft/s; (d) \( 0; \) (e) \( \frac{1}{2} \)
35. \( y = 2(x + 4); y = 2(x - 4) \)
37. \( \frac{4}{5} \) ft/s when the girl is at least 30 ft from the light pole and \( \frac{4}{5} \) ft/s when she is less than 30 ft from the pole.
39. \( \frac{5}{12} \) in.\(^2\) s when the girl is at least 30 ft from the light pole and \( \frac{5}{12} \) in.\(^2\) s when she is less than 30 ft from the pole.
41. (a) \( (n - k)!; \) (b) \(-1.2826 \)
45. \( f(x) = 4(x + 1) \)
47. \( y = 2(x + 4); y = 2(x - 4) \)
49. \( f(x) = 4(x + 1) \)
Answers to Odd-Numbered Problems A-17

31. True 33. True 35. True 37. False

1. (a) $9x^2$; (b) $10x^2 + 3$; (c) $-\frac{1}{3}x^4$; (d) $\frac{6x}{x^2 + 2}^2$
2. (a) $3\cos 3x$; (b) $\frac{x}{\sqrt{x^2 + 5}}$; (c) $-\pi \sin \pi x$
3. (a) $f(x) = 3x$ at $x = 1$; (b) $f(x) = 4x^2$ at $x = 2$;
    (c) $f(x) = \sqrt{x}$ at $x = 3$; (d) $f(x) = \sin x$ at $x = \pi$;
    (e) $f(x) = \tan x$ at $x = \pi$; (f) $f(x) = \frac{1}{\sqrt{x}}$ at $x = 5$
4. (a) $y = 3$; (b) $y = -y$; (c) $-\frac{1}{2} + \frac{2}{x}$
5. $x^2 + 3$; (b) $10x^2 + 3$; (c) $-\frac{1}{3}x^4$; (d) $\frac{6x}{x^2 + 2}^2$

1. Critical points: $-2, 0, 2, 4$; maximum value 10; minimum value 1
3. Critical points: $-2, -1, 0, 1, 2, 3, 4$; maximum value 3; minimum value 1
5. Critical points: $-4, -2, 0$; maximum value 4; minimum value 0
7. Critical points: $-2, -\frac{1}{2}, 1$; maximum value 4, minimum value $-\frac{3}{2}$
9. Critical points: $-1, 1$; No maximum value, minimum value $-1$
11. Critical points: $-1, 3$; No maximum value, no minimum value
13. Critical points: $-2, -1, 0, 1, 2$; maximum value 10; minimum value 1
15. Critical point 0; maximum value 1, no minimum value
17. Critical points: $-\frac{1}{2}, \frac{1}{2}$; Maximum value $\frac{1}{4}$, minimum value $-\frac{1}{4}$
19. Critical points: 0, 1, 3; maximum value 2, minimum value 0
21. Critical points: $-1, 0, 27$; Maximum value 3, minimum value $-1$
23. Critical points: 0, 2, 3, 4, 5, 6, 7, 8; maximum value 1; minimum value $-1$
25. Critical points: $-\frac{\pi}{2}, 0, \frac{\pi}{2}$; maximum value $\frac{\pi^2}{2}$; minimum value 0
27. (a) Critical points: $-1, 2, -\frac{\sqrt{3}}{2}, 2 + \frac{\sqrt{3}}{2}$; maximum value $\approx 2.04$; minimum value $\approx -2.04$
    (b) Critical points: $-1, 0.836, 2 - \frac{\sqrt{3}}{3}, 0.7172, 2 + \frac{\sqrt{3}}{3}, 5$;
    maximum value $\approx 26.04$; minimum value 0
29. Answers will vary. One possibility:

1. Critical points: $-2, 0, 2, 4$; maximum value 10; minimum value 1
3. Critical points: $-2, -1, 0, 1, 2, 3, 4$; maximum value 3; minimum value 1
5. Critical points: $-4, -2, 0$; maximum value 4; minimum value 0
7. Critical points: $-2, -\frac{1}{2}, 1$; maximum value 4, minimum value $-\frac{3}{2}$
9. Critical points: $-1, 1$; No maximum value, minimum value $-1$
11. Critical points: $-1, 3$; No maximum value, no minimum value
13. Critical points: $-2, -1, 0, 1, 2$; maximum value 10; minimum value 1
15. Critical point 0; maximum value 1, no minimum value
17. Critical points: $-\frac{1}{2}, \frac{1}{2}$; Maximum value $\frac{1}{4}$, minimum value $-\frac{1}{4}$
19. Critical points: 0, 1, 3; maximum value 2, minimum value 0
21. Critical points: $-1, 0, 27$; Maximum value 3, minimum value $-1$
23. Critical points: 0, 2, 3, 4, 5, 6, 7, 8; maximum value 1; minimum value $-1$
25. Critical points: $-\frac{\pi}{2}, 0, \frac{\pi}{2}$; maximum value $\frac{\pi^2}{2}$; minimum value 0
27. (a) Critical points: $-1, 2, -\frac{\sqrt{3}}{2}, 2 + \frac{\sqrt{3}}{2}$; maximum value $\approx 2.04$; minimum value $\approx -2.04$
    (b) Critical points: $-1, 0.836, 2 - \frac{\sqrt{3}}{3}, 0.7172, 2 + \frac{\sqrt{3}}{3}, 5$;
    maximum value $\approx 26.04$; minimum value 0
29. Answers will vary. One possibility:
33. Answers will vary. One possibility:

36. Answers will vary. One possibility:

23. Increasing on $(-\infty, -1] \cup [1, \infty)$, decreasing on $[-1, 1]$:
concave up on $\left(-\frac{1}{\sqrt{2}}, 0\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right)$, concave down on $(-\infty, -\frac{1}{\sqrt{2}}) \cup (0, \frac{1}{\sqrt{2}})$

25. Increasing on $[0, \frac{\pi}{2}]$, decreasing on $[\frac{\pi}{2}, \pi]$, concave down on $(0, \pi)$

27. Increasing on $[0, \frac{\pi}{2}]$, decreasing on $(-\infty, 0] \cup [\frac{\pi}{2}, \infty)$, concave up on $(-\infty, -\frac{\pi}{2})$, concave down on $(-\frac{\pi}{2}, 0)$

29. 31. 33.
### 43. (a) No conditions needed;  
(b) \( f(x) > \frac{f'(x)}{g'(x)} g(x) \) for all \( x \);  
(c) No conditions needed

#### 45. (a)

![Graph of a function](image)

(b) \((1.3, 5); (c) (-0.25, 3.1) \cup (6.5, 7]\)

(c) No conditions needed

#### 47.

\([-0.598, 0.680]\]

#### 49. (a) \( \frac{dx}{dt} = kx \), \( k \) a constant;  
(b) \( \frac{dP}{dt} > 0 \)  
(c) \( \frac{dP}{dt} < 0, \frac{d^2P}{dt^2} > 0 \)  
(d) \( \frac{d^2P}{dt^2} = 10 \text{ mph/min} \)  
(e) \( \frac{dx}{dt} \) and \( \frac{d^2x}{dt^2} \) are approaching zero.  
(f) \( \frac{dx}{dt} \) is constant.

#### 51. (a) \( \frac{dc}{dt} > 0, \frac{dc}{dt} > 0 \), where \( C \) is the car’s cost. Concave up.  
(b) \( f'(t) \) is oil consumption at time \( t \). Concave up.  
(c) \( \frac{dP}{dt} > 0, \frac{d^2P}{dt^2} < 0 \), where \( P \) is world population. Concave down.  
(d) \( \frac{d^2\theta}{dt^2} > 0, \frac{d\theta}{dt} > 0 \), where \( \theta \) is the angle that the tower makes with the vertical. Concave up.  
(e) \( P = f(t) \) is profit at time \( t \). Concave down.  
(f) \( R \) is revenue at time \( t \).  

#### 53. \( h(t) = \sqrt{\frac{30000}{t^2} + 27000} - 30 \)

#### 57. (a)

<table>
<thead>
<tr>
<th>Depth</th>
<th>( V )</th>
<th>( A = \Delta V )</th>
<th>( r = \sqrt{\Delta V} )</th>
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<td>1</td>
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<td>4</td>
<td>1.13</td>
</tr>
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<td>6</td>
<td>28</td>
<td>8</td>
<td>1.60</td>
</tr>
</tbody>
</table>

(b)

<table>
<thead>
<tr>
<th>Depth</th>
<th>( V )</th>
<th>( A = \Delta V )</th>
<th>( r = \sqrt{\Delta V} )</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>4</td>
<td>1.13</td>
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<tr>
<td>2</td>
<td>9</td>
<td>5</td>
<td>1.26</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>3</td>
<td>0.98</td>
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<tr>
<td>4</td>
<td>14</td>
<td>2</td>
<td>0.98</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>6</td>
<td>1.38</td>
</tr>
<tr>
<td>6</td>
<td>28</td>
<td>8</td>
<td>1.60</td>
</tr>
</tbody>
</table>

### Critical Points

1. Critical points: 0, 4; local minimum at \( x = 4 \); local maximum at \( x = 0 \)  
2. No critical points; no local minima or maxima on \((0, \frac{\pi}{2})\)  
3. Critical point: 0; local minimum at \( \theta = 0 \)  
4. Critical points: \( -\frac{\pi}{2}, \frac{\pi}{2} \); local minimum at \( x = -\frac{\pi}{2} \); local maximum at \( x = \frac{\pi}{2} \)  
5. Critical point: \( \frac{\pi}{2} \); local minimum value \( f\left(\frac{\pi}{2}\right) = -\frac{\pi}{2} \), no local maximum  
6. Critical points: \( 0, \frac{\pi}{2} \); local minimum value \( f(0) = -\frac{\pi}{2} \), no local maximum  
7. No critical points  
8. No local minimum or maximum values  
9. No critical points  
10. No local minimum or maximum values  
11. Critical points: \( \pm \frac{\pi}{2} \); local minimum value \( f(\pm \frac{\pi}{2}) = -\frac{\pi}{2} \), local maximum value \( f(\pm \frac{\pi}{2}) = 0 \)  
12. Critical points: \(-1, 1\); local minimum value \( f(1) = -1 \), local maximum value \( f(-1) = 1 \)  
13. Critical points \( 0, \frac{\pi}{2} \); local minimum value \( H\left(\frac{\pi}{2}\right) = -\frac{\pi}{2} \), no local maximum  
14. Critical points: \( 0, \frac{\pi}{2} \); local minimum values; local maximum value \( g(\frac{\pi}{2}) = \pi \)  
15. No critical points  
16. No local minimum or maximum values  
17. No critical points  
18. No local minimum or maximum values  
19. Critical point: \( \frac{\pi}{2} \); local minimum value \( f(\frac{\pi}{2}) = 0 \); no local maximum  
20. Maximum value \( f(\frac{\pi}{2}) = 1 \); minimum value \( f(0) = f(\pi) = 0 \)  
21. Maximum value \( g(\frac{\pi}{2}) = \frac{1}{6} \); minimum value \( g(0) = 0 \)  
22. Maximum value \( F(\frac{\pi}{2}) = -\frac{\pi}{2} \); minimum value \( F(0) = 0 \)  
23. Maximum value \( F(\frac{\pi}{2}) = -\frac{\pi}{2} \); minimum value \( F(0) = 0 \)  
24. Maximum value \( H(-1) = H(-1) = 0 \)  
25. Maximum value \( H(-1) = H(-1) = \frac{\pi}{2} \); minimum value \( H(0) = 0 \)  
26. Minimum value \( f(\tan^{-1}(\frac{1}{3})) = 125 \), no maximum value  
27. Maximum value \( H(-1) = H(-1) = 0 \)  
28. Maximum value \( H(-1) = H(-1) = 0 \)  
29. Maximum value \( H(-1) = H(-1) = 0 \)  
30. Local minimum at \( x = 0 \)  
31. Local maximum at \( x = 0 \)  
32. Local minimum at \( x = 0 \)  
33. Local maximum at \( x = 0 \)  
34. No local extrema  
35. No local extrema  
36. Answers will vary. One possibility:
39. Answers will vary. One possibility:

41. Answers will vary. One possibility:

45. $f$ has an inflection point at $c$. 

1. -4 and 4
3. $\left( \frac{3}{\sqrt{2}}, 2 \right)$
5. $\left( \frac{3}{\sqrt{2}}, 2 \right)$
7. $\frac{1}{2}$
9. 1024 in$^3$
11. $x = 10$ ft, $y = 40$ ft
13. $x = 15\sqrt{3}$ ft, $y = 20\sqrt{3}$ ft
15. $x = \frac{10\sqrt{3}}{3}$, $y = 5\sqrt{3}$ ft
17. $P(2\sqrt{2}, 2)$, Q(0, 0)
19. $\frac{6}{\sqrt{2}}$ miles down the shore from $P$
21. At the town
23. about 8:09 A.M.
25. $\frac{5\pi\sqrt{3}}{9}\sqrt{2}$
27. $h = \sqrt{2}r$, $x = \frac{r}{\sqrt{2}}$ where $h =$ height of the cylinder, $x =$ radius of the cylinder, $r =$ radius of the sphere
29. (a) 43.50 cm from one end; shorter length bent to form square
(b) No cut, wire bent to form square
31. height = $\left( \frac{3\sqrt{3}}{\pi} \right)^{1/3}$, radius = $\left( \frac{3\sqrt{3}}{\pi} \right)^{1/3}$
33. $r = \sqrt{A/\theta}$, $\theta = 2$ by $h$
37. $r = \sqrt{A/(6\pi)}$, $h = 2r$
39. Maximum area is for a square.
41. $\pi/3$
43. $x = 1$, $y = 3$, $z = 3$
45. (a) $x = 2a/3$ maximizes area of $A$.
(b) $x = 2a/3$ minimizes area of $B$.
(c) $x = 2a/3$ minimizes length $z$.
47. (a) $L' = 3$, $L = 4$, $\phi = 90^\circ$; (b) $L' = 5$, $L = 12$, $\phi = 90^\circ$;
(c) $\phi = 90^\circ$, $L = \sqrt{2l^2 - l^2}$, $L' = h$
51. (a) Not possible; (b) Not possible;

Global minimum: \( f(\frac{-\pi}{2}) = -2 \)
Global maximum: \( f(\frac{\pi}{2}) = 2 \)
Inflection points: \( (-\frac{\pi}{2}, -2), (-\frac{\pi}{2}, -2) \)

(b) 
Global minimum: \( f(\frac{-\pi}{2}) = -1 \)
Global maximum: \( f(\frac{\pi}{2}) = 3 \)
Inflection points: \( (\frac{\pi}{2}, 3), (\frac{\pi}{2}, 3) \)

55. (a) Increasing on \((-\infty, -3] \cup [0, \infty)\); decreasing on \([-3, -1) \cup (1, \infty)\);

(b) Concave up on \((-2, 0) \cup (0, 2)\); concave down on \((-\infty, -2) \cup (2, \infty)\);

(c) Local maximum at \(x = -3\); local minimum at \(x = 1\);

(d) \(x = -1.2\)

Global minimum: \( f(\frac{-\pi}{2}) \approx -1.9 \)
Global maximum: \( f(0.97) \approx 1.9 \)
Inflection points: \( (-\frac{\pi}{2}, 0), \left(\frac{\pi}{2}, 0\right), \left(-2.469, 0.542\right), \left(-0.673, -0.542\right), \left(0.413, 0.408\right), \left(2.729, -0.408\right) \)

57. 
Global minimum: \( f(-1) \approx -4.9 \)
Global maximum: \( f(7) = 48.0 \)
Inflection point: \( (2.02, 11.4) \)

59. (a) 
Global minimum: \( f(0) = 0 \)
Global maximum: \( f(7) \approx 124.4 \)
Inflection point: \( (2.34, 48.09) \)
1. \( c < 2 \)  
3. \( c = 0 \)  
5. \( c = -1 \)  
7. \( c = 1 \)  
9. \( c = 3 - \sqrt{3} \approx 1.27 \)  
11. \( c = \frac{1}{3} \approx 0.59 \)  
13. \( c = \left(\frac{2}{3}\right)^{\frac{1}{2}} \approx 0.46 \)  
15. \( c = \pm \frac{5}{2} \)  
17. \( \text{Does not apply. } T(\theta) \)  
19. \( c = \sqrt{2} \approx 1.41 \)  
21. \( \text{Does not apply. } f(x) \text{ is not differentiable at } x = 0 \)  
23. \( \approx 1.5, 3.75, 7 \)  
1. \( 1.66 \)  
3. \( 1.11 \)  
5. \( -0.12061 \)  
7. \( 3.69815 \)  
9. \( 0.45018 \)  
11. \( 2.05879, 3.41421 \)  
13. \( 0.48909 \)  
15. \( 1.83712 \)  
17. \( \text{Minimum } f(-0.60583) \approx -0.32645; \text{ Maximum } f(1) = 4 \)  
19. \( \text{Minimum } f(1.49349) \approx -0.21723 \)  
21. \( 0.9443 \)  
23. \( (c) \)  
25. \( 0.91486 \)  
27. \( 2.21756 \)  
29. (a)  
(b) \( 0.5 \); (c) \( \frac{1}{2} \)  
31. (a) \( x_1 = 0, x_2 = 1, x_3 = 1.4142136, x_4 = 1.533774, x_5 = 1.5980532 \)  
(b) \( x = \left(1 + \sqrt{5}\right) \approx 1.618034 \)  
33. (a) \( x_1 = 1.5, x_2 = 2.5, x_3 \approx 1.6666667, x_4 = 1.6 \)  
(b) \( x = \frac{1 + \sqrt{5}}{2} \approx 1.618034 \)  
(c) \( (1 + \sqrt{5})/2 \approx 1.618034 \)  
35. (a) \( \text{The algorithm computes the root of } x - a = 0 \) for \( x_i \) close to \( \frac{1}{\sqrt{2}} \)  
37. \( 20.84 \) ft.  
39. (a) \( (2.86729, 3.1838) \)  
(b) \( (0.67278, 4.1031) \)
A-24 Answers to Odd-Numbered Problems

39. \( \frac{3}{5}x^{10} + C \) if \( x > 0 \)
41. \( \frac{1}{2}x^3 + \frac{1}{3} + C \) if \( x > 0 \)
45. \( x^3\sqrt{x - 1} + C \)
47. \( \frac{5x^2}{2} + C \)
51. \( x^4 + C \) if \( x > 0 \), \(-x^2 + C \) if \( x < 0 \)
53. (a) \(-2\cos(3(x - 2)) + C \)
(b) \( \frac{1}{2}\cos\frac{\pi}{2} - 2\sin\frac{\pi}{2} + C \)
(c) \( \frac{1}{2}x^2\sin 2x + C \)
5. \( y = \frac{1}{2}x^3 + x + C \)
7. \( y = x^\sqrt{2} + C \)
9. \( z = \frac{x}{x^2 + 1} \)
11. \( x = \frac{\sqrt{3}y^3 + 2\sqrt{y} - x + C \quad \frac{x}{y^4 + 2\sqrt{y} - x + 100} \)
13. \( y = \frac{1}{2}(2x + 1)^3 + C \quad y = \frac{1}{2}(2x + 1)^3 + \frac{1}{2} \)
15. \( y = \frac{x}{x^2 + 1} \)
17. \( v = 5 \text{ cm/s} \quad \frac{7}{13} \)
19. \( v = 2.83 \text{ cm/s} \quad \frac{7}{13} \)
21. \( 1.44 \text{ ft} \)
23. \( t = 0.66, 1.75 \text{ s} \quad \frac{7}{13} \)
27. Moon: \( v = 1.470 \text{ mi/s} \quad \frac{7}{13} \)
Jupiter: \( v = 36.812 \text{ mi/s} \quad \frac{7}{13} \)
Sun: \( v = 382.908 \text{ mi/s} \quad \frac{7}{13} \)
29. \( 2.2 \text{ ft/s}^3 \quad \frac{7}{13} \)
31. \( 5500 \text{ m} \quad \frac{7}{13} \)
33. (a) \( \frac{dV}{dT} = \frac{V}{10} \quad \frac{7}{13} \)
(b) \( V = \frac{1}{10}(-20t + 900)^2 \quad \frac{7}{13} \)
(c) \( V = 1.5 \text{ cm}^3 \quad \frac{7}{13} \)
35. (a) \( \frac{dV}{dt} = \frac{C}{\sqrt{V}} \quad V(0) = 1600, V(46) = 0 \quad \frac{7}{13} \)
(b) \( V = \frac{1}{10}(-20t + 960)^2 \quad V(0) = 600 \quad \frac{7}{13} \)
37. (a) \( t(t) = \begin{cases} -32t & \text{for } 0 \leq t < 1 \\ -32(t - 1) + 24 & \text{for } 1 \leq t < 2.5 \end{cases} \quad \frac{7}{13} \)
(b) \( t = 0.66, 1.75 \text{ s} \quad \frac{7}{13} \)
41. True 43. True 45. False 47. True

1. Critical points: 0, 1, 4; minimum value \( f(1) = -1 \); maximum value \( f(4) = 8 \)
3. Critical points: -2, -\( \frac{3}{2} \); minimum value \( f(-2) = \frac{3}{2} \); maximum value \( f(-\frac{3}{2}) = 4 \)
5. Critical points: -\( \frac{1}{2} \), 0, 1; minimum value \( f(0) = 0 \); maximum value \( f(1) = 1 \)
7. Critical points: -2, 0, 1, 3; minimum value \( f(1) = -1 \); maximum value \( f(3) = 135 \)
9. Critical points: -1, 0, 2, 3; minimum value \( f(2) = -9 \); maximum value \( f(3) = 88 \)
11. Critical points: \( \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \); minimum value \( f(\frac{3}{2}) = -0.87 \); maximum value \( f(\frac{5}{2}) = 1 \)
13. Increasing: \( (-\infty, \frac{1}{2}) \); concave down: \( (-\infty, \infty) \)
15. Increasing: \( (-\infty, -1) \cup [1, \infty) \); concave down: \( (-\infty, 0) \)
17. Increasing: \( (0, \infty) \); concave down: \( (-\infty, 0) \)
19. Increasing: \( (-\infty, \frac{1}{2}) \); concave down: \( (-\infty, 0) \cup (\frac{1}{2}, \infty) \)
21. Increasing: \( (-\infty, 0) \cup [\frac{1}{3}, \infty) \); decreasing: \( [0, \frac{1}{3}) \)
Local minimum value \( f\left(\frac{1}{3}\right) = -\frac{1}{3} \)
Local maximum value \( f(0) = 0 \)
Inflation point: \( (\frac{1}{3}, -\frac{1}{3}) \)
43. $r = 4\sqrt{2}$, $h = 8\sqrt{2}$

45. (a) $c = \pm \sqrt{3}$  
(b) Does not apply. $F'(0)$ does not exist.

49. $A = 6$

51. $A = \frac{100}{3}$

53. $\frac{5}{2}$

55. $4$

57. $\frac{1}{2}$

59. $2\frac{1}{2}$ ft

63. (a) $\frac{1}{6}$; (b) 31; (c) 39

65. (a) 4; (b) $\frac{51}{5}$; (c) 10.5; (d) 102.4

1. $5.625$

3. $15.6875$

5. $2.625$

7. $\int \frac{x^2}{1 + x^4} \, dx$

9. $\int \frac{x^2}{1 + x^4} \, dx$

11. $4$

13. $2\pi - 3$

15. $\frac{11}{7}$

17. $\frac{17}{2}$

19. $\frac{1}{2} + \frac{1}{4}$

21. $\frac{1}{2} = A^2$

23. $\frac{1}{2}$

25. 3

27. 40, 80, 120, 160, 200, 240

29. 20, 80, 160, 240, 320, 400

31. (a) $-3$; (b) 19; (c) 5; (d) 2; (e) 9; (f) 0

35. Left: 5.24; Right: 6.84; Midpoint: 5.98

37. Left: 0.8638; Right: 0.8178; Midpoint: 0.8418

1. $A(x) = 2x$

3. $A(x) = \frac{1}{2}(x - 1)(-1 + x), x > 1$

5. $A(x) = ax^2/2$
A-26 Answers to Odd-Numbered Problems

7. \( A(x) =
\begin{cases} 
2x & \text{if } 0 \leq x \leq 1 \\
2 + (x - 1) & \text{if } 1 < x \leq 2 \\
3 + 2(x - 2) & \text{if } 2 < x \leq 3 \\
5 + (x - 3) & \text{if } 3 < x \leq 4 \\
\text{etc.}
\end{cases}
\)

11. \( f(x) = 2x + 3(x - 2) \cos 2x + 2x \sin(x^2) \)

13. \( f(x) \) is increasing on \((0, \infty)\) and concave up on \((0, \infty)\).

17. \( f(x) \) is increasing on \((0, \infty)\) and never concave up.

27. \( f(x) \) is increasing on \((0, \infty)\) and concave up on \((\pi, 2\pi), (3\pi, 4\pi), \ldots\).

33. \( f(x) \) is increasing on \((0, \infty)\) and concave up on \((\pi, 2\pi), (3\pi, 4\pi), \ldots\).

37. \( f(x) \) is increasing on \((0, \infty)\) and concave up on \((\pi, 2\pi), (3\pi, 4\pi), \ldots\).

47. Lower bound 20.

49. \( \frac{3}{4} \) 51. \( 2 \) 53. \( \sqrt{2} \) 55. True 57. False

59. True

61. \( f(x) = \frac{1}{4}x^2 - \frac{1}{2}x^2, \quad 0 \leq t \leq 2 \)

62. \( f(x) = \frac{1}{4}x^2 - \frac{1}{2}x^2, \quad t > 2 \)

\( t = 4 + 2\sqrt{2} \approx 6.83 \)

1. \( \frac{1}{2}(2x + 2)^{\frac{3}{2}} + C \)

3. \( \sin(2x + 2) + C \)

5. \( \frac{1}{2}(2x + 2)^{\frac{3}{2}} + C \)

7. \( \frac{1}{2}(2x + 2)^{\frac{3}{2}} + C \)

9. \( \cos(2x + 2) + C \)

11. \( \sin(2x + 2) + C \)

13. \( \cos(2x + 2) + C \)

15. \( \frac{1}{2}(2x + 2)^{\frac{3}{2}} + C \)

17. \( \sin(2x + 2) + C \)

19. \( \frac{1}{2}(2x + 2)^{\frac{3}{2}} + C \)

21. \( \sin(2x + 2) + C \)

23. \( \frac{1}{2}(2x + 2)^{\frac{3}{2}} + C \)

25. \( \cos(2x + 2) + C \)

27. \( \cos(2x + 2) + C \)

29. \( \cos(2x + 2) + C \)

31. \( \cos(2x + 2) + C \)

33. \( \cos(2x + 2) + C \)

35. \( \cos(2x + 2) + C \)

37. \( \cos(2x + 2) + C \)

39. \( \cos(2x + 2) + C \)

41. \( \cos(2x + 2) + C \)

43. \( \cos(2x + 2) + C \)

45. \( \cos(2x + 2) + C \)

47. \( \cos(2x + 2) + C \)

49. \( \cos(2x + 2) + C \)

51. \( \cos(2x + 2) + C \)

53. \( \cos(2x + 2) + C \)

55. \( \cos(2x + 2) + C \)

57. \( \cos(2x + 2) + C \)

59. \( \cos(2x + 2) + C \)

61. \( \cos(2x + 2) + C \)

63. \( \cos(2x + 2) + C \)

65. \( \cos(2x + 2) + C \)

67. \( \cos(2x + 2) + C \)

69. \( \cos(2x + 2) + C \)

71. \( \cos(2x + 2) + C \)

73. \( \cos(2x + 2) + C \)

75. \( \cos(2x + 2) + C \)

77. \( \cos(2x + 2) + C \)

79. \( \cos(2x + 2) + C \)

81. \( \cos(2x + 2) + C \)

83. \( \cos(2x + 2) + C \)

85. \( \cos(2x + 2) + C \)

87. \( \cos(2x + 2) + C \)

89. \( \cos(2x + 2) + C \)

91. \( \cos(2x + 2) + C \)

93. \( \cos(2x + 2) + C \)

95. \( \cos(2x + 2) + C \)

97. \( \cos(2x + 2) + C \)

99. \( \cos(2x + 2) + C \)

101. \( \cos(2x + 2) + C \)

103. \( \cos(2x + 2) + C \)

105. \( \cos(2x + 2) + C \)

107. \( \cos(2x + 2) + C \)

109. \( \cos(2x + 2) + C \)

111. \( \cos(2x + 2) + C \)

113. \( \cos(2x + 2) + C \)

115. \( \cos(2x + 2) + C \)

117. \( \cos(2x + 2) + C \)

119. \( \cos(2x + 2) + C \)

121. \( \cos(2x + 2) + C \)

123. \( \cos(2x + 2) + C \)

125. \( \cos(2x + 2) + C \)

127. \( \cos(2x + 2) + C \)

129. \( \cos(2x + 2) + C \)

131. \( \cos(2x + 2) + C \)

133. \( \cos(2x + 2) + C \)

135. \( \cos(2x + 2) + C \)

137. \( \cos(2x + 2) + C \)

139. \( \cos(2x + 2) + C \)

141. \( \cos(2x + 2) + C \)

143. \( \cos(2x + 2) + C \)

145. \( \cos(2x + 2) + C \)

147. \( \cos(2x + 2) + C \)

149. \( \cos(2x + 2) + C \)

151. \( \cos(2x + 2) + C \)
1. 0.7877, 0.5654, 0.6766, 0.6671

2. 1.6847, 2.0382, 1.8615, 1.8755

3. 3.4966, 7.4966, 5.4966, 5.2580

4. 0.4892, 0.4392, 0.4634, 0.4642

5. 4570 ft

6. 1.074, 4.585, 600 ft

7. Using a right Riemann sum, the result is 13,740 gallons.

8. True

9. True

10. True

11. True

12. True

13. True

14. True

15. False

16. False

17. True

18. True

19. True

20. True

21. True

22. True

23. True

24. True

25. True

26. True

27. True

28. True

29. False

30. True

31. 130 ft; 194 ft

32. 6 s, 2 + 2√2 s

33. Area (A) = 9; A(B) = 4\(\pi\); A(C) = 12; A(D) = 56; A(A + B + C + D) = 36
A-28 Answers to Odd-Numbered Problems

1. \( \frac{2\pi}{3} \)
2. \( \frac{8\pi}{3} \)

3. (a) \( \frac{2\pi}{3} \) (b) \( 8\pi \)

4. \( \frac{4\pi}{3} \)

5. (a), (b)
   (c) \( \Delta V = 2\pi \left[ (5\pi^{3/2} - 5\pi^{1/2}) \right] d\pi 
   (d) \( 2\pi \int_{\pi}^{5\pi} (5\pi^{3/2} - 5\pi^{1/2}) \, d\pi 
   (e) \frac{40\sqrt{\pi}}{3} \)

7. (a), (b)
   (c) \( \Delta V = 2\pi \left[ \frac{1}{2} x^2 + x^2 \right] dx \)
   (d) \( 2\pi \int_{1}^{3} \left( \frac{1}{2} x^2 + x^2 \right) \, dx \)
   (e) \( \frac{3\pi^2}{2} \)

9. (a), (b)
   (c) \( \Delta V = 2\pi y^3 dy \)
   (d) \( 2\pi \int_{0}^{1} y^3 \, dy \)
   (e) \( \frac{\pi}{4} \)

11. (a), (b)
   (c) \( \Delta V = 2\pi (2y^2 - y^3) dy \)
   (d) \( 2\pi \int_{0}^{1} (2y^2 - y^3) \, dy \)
   (e) \( \frac{\pi}{6} \)

13. (a) \( \pi \int_{a}^{b} [f(x)]^2 \, dx \)
   (b) \( 2\pi \int_{a}^{b} [f(x) - g(x)] \, dx \)
   (c) \( 2\pi \int_{a}^{b} [f(x) - g(x)] \, dx \)
   (d) \( 2\pi \int_{a}^{b} [f(x) - g(x)] \, dx \)

17. \( \frac{4\pi^2}{3} \)

19. \( \frac{4\pi}{3}(b^2 - a^2)^{3/2} \)

23. (a) \( \frac{\pi}{2} \) (b) \( \frac{\pi}{2} \) (c) \( \frac{\pi}{2} \)

25. \( \frac{1}{2} \)
1. \( \frac{1}{2}(181\sqrt{11} - 13\sqrt{13}) \)  
3. 9  
5. \( \frac{33}{2} \)  
7. \( [2\sqrt{2} - 1] \)  
9. 4\( \pi \)  
11. \( 2\sqrt{2} \)  
13. \( \int_{0}^{\pi/2} \sqrt{\cos^2 \theta + 4 \sin^2 \theta} \, d\theta = 4.6468 \)  
15. \( \int_{0}^{\pi/2} \sqrt{\cos^2 \theta + 4 \sin^2 \theta} \, d\theta = 2.3241 \)  
17. 6\( \pi \)  
19. 6\( \pi \)  
21. (a) \( \left(4\sqrt{2} - 1\right) \); (b) 16  
23. \( \frac{6\sqrt{3}}{17} \pi \)  
25. \( 248\sqrt{2}\pi/9 \)  
27. \( \frac{9}{10\sqrt{15} - 1} \)  
29. \( \frac{3}{4}\pi \)  
33. (a)  
(b) \( 2\pi \)  
35. (a)  
(b)  
(c)  
(d)  
19. \( \frac{3}{4}\pi \)  
21. (a) \( \left(4\sqrt{2} - 1\right) \); (b) 16  
23. \( \frac{6\sqrt{3}}{17} \pi \)  
25. \( 248\sqrt{2}\pi/9 \)  
27. \( \frac{9}{10\sqrt{15} - 1} \)  
29. (a) \( V = 2\pi \int_{0}^{1} (K - \gamma) w(\rho) \, d\gamma \)
31. (a) \(4\pi r^3 n \sin \frac{\pi r}{R} \cos^2 \frac{\pi r}{a}\)  
35. \(\bar{x} \approx 7.00\) cm above the center of the hole; \(\bar{y} \approx 0.669\) cm to the right of the center of the hole

Problem Set 8.8

1. (a) 0.1  (b) 0.35
2. (a) 0.2  (b) 0
3. (a) 0.6  (b) 2.2
4. (a) 0.6  (b) 2

9. (a) 0.9  (b) 10  (c) \(F(x) = \begin{cases} 
0, & x < 0 \\
\frac{x}{20}, & 0 \leq x \leq 20 \\
1, & x > 20 
\end{cases}\)

11. (a) \(\frac{27}{32}\)  (b) 4
   (c) \(F(x) = \begin{cases} 
0, & x < 0 \\
\frac{3}{64}x^2 - \frac{1}{256}x^3, & 0 \leq x \leq 8 \\
1, & x > 8 
\end{cases}\)

13. (a) 0.6875  (b) 2.4
   (c) \(F(x) = \begin{cases} 
0, & x < 0 \\
\frac{x}{16} - \frac{3}{256}x^4, & 0 \leq x \leq 4 \\
1, & x > 4 
\end{cases}\)

15. (a) \(\frac{1}{2}\)  (b) 2
   (c) \(F(x) = \begin{cases} 
0, & x < 0 \\
\frac{1}{2} - \frac{1}{2}\cos \frac{x\pi}{4}, & 0 \leq x \leq 4 \\
1, & x > 4 
\end{cases}\)

17. (a) \(\frac{1}{3}\)  (b) \(\frac{4}{3}\ln 4\)  (c) \(F(x) = \begin{cases} 
0, & x < 1 \\
\frac{4x - 4}{3x}, & 1 \leq x \leq 4 \\
1, & x > 4 
\end{cases}\)

21. \(\frac{a + b}{2}\)  23. \(k = \frac{6}{125}\)

33. \(F(x) = \begin{cases} 
0, & x < 0 \\
0.8, & 0 \leq x < 1 \\
0.9, & 1 \leq x < 2 \\
0.95, & 2 \leq x < 3 \\
1, & x \geq 3 
\end{cases}\)

35. (a) 1  (b) \(\frac{1}{12}\)  (c) \(\frac{2}{(y + 1)^2}\) for \(0 \leq y \leq 1\)  (d) 0.38625

37. \(\frac{32}{7}\)  39. \(\frac{4}{7}\)

Chapter Review 8.8

21. True  23. True

Sample Test

1. \(\frac{1}{5}\)  3. \(\frac{\pi}{6}\)  5. \(\frac{5\pi}{6}\)
7. \(V(S_1) = \frac{\pi}{30}; V(S_2) = \frac{\pi}{6}; V(S_3) = \frac{\pi}{10}; V(S_4) = \frac{5\pi}{6}\)
9. 205,837 ft-lb  11. (a), (b) \(\frac{32}{3}\)  13. \(\frac{3\pi\pi}{15}\)
15. \(\frac{53}{6}\)  17. 36  19. \(\pi \int_a^b \left[f(x) - g^2(x)\right] dx\)
21. \(M_y = \delta \int_a^b x[f(x) - g(x)] dx\)
25. (a) \( \frac{1}{4} \)  (b) \( \frac{1}{8} \)  (c) 2

(d) \( F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{x^3}{8} + x - 1, & \text{if } 0 \leq x \leq 2 \\ \frac{x^2}{8} + x - 1, & \text{if } 2 < x \leq 4 \\ 1, & \text{if } x > 4 \end{cases} \)

(e) \( F(y) = \begin{cases} 0, & \text{if } y < 0 \\ \frac{y^2}{28800}, & \text{if } 0 \leq y \leq 120 \\ \frac{y^2}{28800} + \frac{y}{60} - 1, & \text{if } 120 < y \leq 240 \\ 1, & \text{if } y > 240 \end{cases} \)

27. (a) \( 95,802,719 \)  (b) \( 0.884 \)  (c) 0.2625

(d) For 0 ≤ x ≤ 0.6, \( F(x) \approx 6.3868 \times 10^6 x^{15} - 3.2847 \times 10^7 x^{14} + 7.4284 \times 10^7 x^{13} - 9.6569 \times 10^7 x^{12} + 7.9011 \times 10^7 x^{11} - 4.1718 \times 10^7 x^{10} + 1.3906 \times 10^7 x^9 - 2.6819 \times 10^6 x^8 + 5.9897 \times 10^6 x^7 \)

(e) \( F(25.4y) \) where \( F \) is as in (d)

29. \( G(y) = \begin{cases} 0, & y < 0 \\ \sqrt{y^2 - 1}, & 0 \leq y \leq \sqrt{2} \\ 1, & y > \sqrt{2} \end{cases} \)

\( g(y) = y/\sqrt{y^2 - 1}, 0 \leq y \leq \sqrt{2} \)

\[ M_x = \frac{1}{2} \int_0^b \left[ f^2(x) - g^2(x) \right] \, dx \]

\[ 2\pi \int_a^b f(x) \sqrt{1 + [f'(x)]^2} \, dx \]

\[ + 2\pi \int_a^b g(x) \sqrt{1 + [g'(x)]^2} \, dx \]

\[ + \pi[f'(a) - g'(a)] + \pi[f'(b) - g'(b)] \]

25. (a) \( \frac{3}{4} \)  (b) \( \frac{6 - x}{18} \) for 0 ≤ x ≤ 6  (c) 2

1. \( - \frac{1}{x} + C \)  3. \( - \frac{100}{x^{0.01}} + C \)  5. 0  7. \( \frac{2}{x} \)

9. (a) 2; (b) 2.48832; (c) 2.593742; (d) 2.691588; (e) 2.704814

11. (a) 2.25; (b) 2.593742; (c) 2.6533; (d) 2.70481; (e) 2.71152

13. \( x = \frac{12k + 1}{6} \pi \) or \( x = \frac{12k + 5}{6} \pi \)  15. \( x = \frac{4k + 1}{4} \pi \)
17. \ \sin \theta = \frac{\sqrt{x^2 - 1}}{x}, \ \cos \theta = \frac{1}{\sqrt{x^2 - 1}}, \ \tan \theta = \sqrt{x^2 - 1} \\
\ \cot \theta = \frac{1}{\sqrt{x^2 - 1}}, \ \sec \theta = x, \ \csc \theta = \frac{1}{x}, \ \sec \theta = \sqrt{x^2 - 1} \\
\ \cot \theta = x, \ \sec \theta = \frac{\sqrt{x^2 + 1}}{x}, \ \csc \theta = \sqrt{1 + \frac{1}{x^2}} \\
21. \ y = -\frac{2}{x^2 - 2} \\

1. (a) 1.792; (b) 0.406; (c) 3.871; (d) 0.3465; (e) -3.584; (f) 3.871 \\
3. \frac{2x + 3}{x - 2} \\
5. \frac{3}{x - 3} \\
9. \frac{2x + 4x \ln x + \frac{1}{x} (\ln x)^2}{\sqrt{x^2 + 1}} \\
13. \left\{ \begin{array}{l} \frac{1}{2} \ln(2x + 1) + C \\
\frac{1}{3} \ln(3\theta + 9\theta^2) + C \\
\frac{1}{2} \ln(4x + 4) - \ln x \end{array} \right. \\
15. \left\{ \begin{array}{l} \frac{1}{2} \ln(2x + 1) + C \\
\frac{1}{3} \ln(3\theta + 9\theta^2) + C \\
\frac{1}{2} \ln(4x + 4) - \ln x \end{array} \right. \\
23. \left\{ \begin{array}{l} \frac{1}{2} (x + \ln x - 1) + C \\
\frac{1}{2} (x + \ln x - 1) + C \\
\frac{1}{2} (x + \ln x - 1) + C \end{array} \right. \\
25. \left\{ \begin{array}{l} \frac{1}{4} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \\
\frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \\
\frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \end{array} \right. \\
27. \left\{ \begin{array}{l} \frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \\
\frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \\
\frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \end{array} \right. \\
31. \left\{ \begin{array}{l} \frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \\
\frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \\
\frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \end{array} \right. \\
33. \left\{ \begin{array}{l} \frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \\
\frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \\
\frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \end{array} \right. \\
35. \left\{ \begin{array}{l} \frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \\
\frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \\
\frac{1}{6} (x + 1)^3 + 8x^2 - 64x + 256 \ln(2x + 4) + C \end{array} \right. \\
41. \text{Minimum } f'(1) = -1 \\
43. \lim_{x \to \infty} \ln x = -\infty \\
45. \ x = 3 \\
47. \ln 2 \\
49. \ln \sqrt{3} = 0.5493 \\
51. \ln \sqrt{3} = 0.5493 \\
53. \ln \sqrt{4} = 3.35 \\
55. \text{Maxima: } \left( \frac{1}{2}, 0.916 \right), \left( \frac{1}{2}, 0.916 \right), \text{ minimum: } \left( \frac{1}{2}, -0.693 \right) \\
57. \text{Maximum: } \left( \frac{1}{2}, 0.916 \right), \left( \frac{1}{2}, 0.916 \right), \text{ minimum: } \left( \frac{1}{2}, -0.693 \right) \\
59. \left\{ \begin{array}{l} \text{Minimum: } \left( \frac{1}{2}, 0.139 \right); \\
\text{Maximum: } \left( \frac{1}{2}, 0.260 \right) \end{array} \right.
27. Domain = \((\infty, 0)\); increasing on \((-\infty, 1)\) and decreasing on \((1, \infty)\); concave up on \((2, \infty)\) and concave down on \((-\infty, 2)\); point of inflection at \((2, 2/e^2)\)

29. Domain = \((-\infty, \infty)\); increasing on \((-\infty, 0)\); concave up on \((-\infty, 0)\); no extreme values or points of inflection.

31. Domain = \((-\infty, \infty)\); increasing on \((-\infty, \infty)\); concave upward on \((-\infty, \infty)\); no extreme values or points of inflection.

33. Domain = \((-\infty, \infty)\); increasing on \((-\infty, 2)\) and decreasing on \((2, \infty)\); maximum at \((2, 1)\); concave up on \((2, 2)\) and concave down on \((-\infty, 2)\); points of inflection at \((2, 1)\) and \((2, 2)\)

35. Domain = \((-\infty, \infty)\); increasing on \((-\infty, 0)\); concave up on \((-\infty, 0)\) and concave down on \((0, \infty)\); point of inflection at \((0, 0)\); no extreme values.

37. Domain = \((-\infty, \infty)\); increasing on \((0, \infty)\) and decreasing on \((-\infty, 0)\); concave up on \((-\infty, 0)\) and concave down on \((0, \infty)\); point of inflection at \((0, 0)\); no extreme values.

39. \(\frac{1}{2}e^{3x} + C\)

41. \(e^{-x} + C\)

43. \(\frac{3}{2}e^{2x} \ln(2) - 1\)

45. \(\frac{3}{2}x + C\)

47. \(\frac{3}{2}x + C\)

49. (a) 3.628800; 3.598696; (b) 8.31 \times 10^7

51. \(\sqrt{2}(e - 1)\)

53. (a) 0.0 (b) Maximum: \(\left(\frac{1}{2}, \frac{1}{2}\right)\); minimum: \(\left(-\frac{1}{2}, -\frac{1}{2}\right)\); (c) \(\sqrt{e}\)

55. (a) 3.11; (b) 0.911

57. 4.264

59. Behaves like \(-x\); behaves like \(2 \ln x\)

1. \(3\)
2. \(3\)
3. \(5\)
4. \(9\)
5. \(7\)
6. \(9\)
7. \(1.544\)
8. \(0.1747\)
9. \(1.9307\)
10. \(1.9307\)
11. \(1.9307\)
12. \(1.9307\)
13. \(1.36866\)
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95. \(1.36866\)
96. \(1.36866\)
97. \(1.36866\)
98. \(1.36866\)
99. \(1.36866\)
100. \(1.36866\)
39. Domain = \((-\infty, \infty)\); increasing on \((-\infty, 0)\) and concave down on \((0, \infty)\); no extreme values; point of inflection at \((0, -0.82)\)

41. \(\log_{10} x = -\log_{10} x\)

43. \(E = 5.017 \times 10^6 \text{ kW-h for magnitude 7.}\) 
\[ E = 1.560 \times 10^8 \text{ kW-h for magnitude 9.}\]

45. \(x = 2^{1/2} = 1.0986; \) frequency of \(C = 440 \sqrt{2} = 523.25\) Hz.

47. If \(y = \frac{a}{x} + b, \) then \(\ln y = \ln a + \ln x + \ln b, \) so the \(\ln y \text{ vs. } x\) plot will be linear. If \(y = C \cdot x^n, \) then \(\ln y = \ln C + d \ln x, \) so the \(\ln y \text{ vs. } \ln x\) plot will be linear.

49. \(f'(x) = x^{(2x \ln x + x)}\)
\(g'(x) = x^{(2x \ln x + x)}\)

55. Exponential growth: 6.93 billion in 2010; 10.29 billion in 2040; 19.92 billion in 2090; Logistic growth: 7.13 billion in 2010; 10.90 billion in 2040; 15.15 billion in 2090

57. \(b = 2 \ln 2, \) \(C = 2 \ln 2\)

59. \(\lim_{x \to \infty} x^x = 1; \) maximum: \((\ln x, e^{1/2})\)

65. \(\lim_{x \to 0} x^x = 1; \) \(y(2) = 10.5\)
3. \( \lim_{y(2)} y(t) = 0 \) and \( y(2) \approx 6 \)

8. The oblique asymptote is \( y = x \).

7. \( y = \frac{1}{2} e^{2t} \)

9. \( y = x + 1 + 3e^{-5} \).

11. Euler's Method \( y_n \)

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13. Euler's Method \( y_n \)

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15. Euler's Method \( y_n \)

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19. (a) \( y(t_1) = 0 \) (b) \( y(t_2) = 0.00099998 \)
(c) \( y(t_3) \approx 0.269097 \)

21. (a) \( \Delta x = \frac{1}{2} f(x_{n_1}, y_{n_1}) + \frac{1}{2} f(x_{n_2}, y_{n_2}) \)
(b) \( x_{n+1} = x_n + h \)
(c) \( y_{n+1} = y_n + h f(x_n, y_n) \)

23. Euler's Method \( y_n \)

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25. Euler's Method \( y_n \)

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27. Euler's Method \( y_n \)

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1. \( \frac{5}{3} \) 3. \( \frac{2}{3} \) 5. \( \frac{3}{5} \) 7. \( \frac{4}{3} \) 9. \( 0.4507 \) 11. \( 0.1115 \)

13. 0.9548 15. 2.056 17. 0.6259 19. \( \sin \frac{1}{2} \) 21. \( a = \sin \frac{1}{2} \) 23. \( \theta = \tan \frac{1}{2} \) 25. \( \frac{1}{3} \) 27. \( \frac{4}{3} \)

33. (a) \( \frac{1}{2} \) (b) \( \frac{1}{2} \)
35. (a) \( \frac{1}{2} \) (b) \( -\frac{1}{2} \)

37. The tangent lines approach the vertical

39. \( \cos x \) 41. \( \sec x \) 43. \( \sqrt{1 - 4x^2} \)
Answers to Odd-Numbered Problems  A-35

45. \[
\frac{1}{2} + \frac{\tan^{-1}(e^x)}{3} + 3 \tan^{-1}(e^x)
\]
47. \[
\frac{3 \tan^{-1} x^2}{3} + \frac{x}{x^2} + C
\]

49. \[
\frac{1}{2} \left( x^2 + 1 \right)
\]

51. \[
\frac{3 (1 + \sin^2 x)}{x} + C
\]

53. \[
\frac{1}{2} \left( 1 + \ln x^2 \right)
\]

55. \[
\sin 2x + C
\]

57. \[
\sin^2 2x + C
\]

59. \[
\frac{1}{2} (x^2 - \sin x)
\]

61. \[
\frac{1}{3} (\tan^{-1} \frac{1}{x})^2 + C
\]

63. \[
\frac{1}{2} \left( x - \frac{3}{2} \right) + C
\]

65. \[
\frac{1}{2} \left( \sec 2x + C \right)
\]

67. \[
\tan^{-1} \left( \frac{\sqrt{x^2 + 1}}{3} \right) + C
\]

69. \[
\frac{1}{2} \tan^{-1} \left( \frac{x - \frac{3}{2}}{2} \right) + C
\]

71. \[
\cot \left( \frac{1}{2} \left( x + 3 \right) \right) + C
\]

73. \[
\frac{\tan \left( \frac{1}{2} x \right)}{2} + C
\]

77. \[
\frac{1}{2} (\sin^2 \frac{1}{2} x)
\]

79. \[
\frac{1}{2} \left( \arcsin x = \arccos x \right)
\]

87. \[
\sin 3x + C
\]

89. \[
\frac{1}{3} \sin 3x
\]

91. \[
\frac{1}{2} \sin 3x + C
\]

93. \[
\frac{1}{2} \sin x
\]

95. \[
\frac{1}{2} \sin x
\]

97. \[
\frac{1}{2} \sin x
\]

99. \[
\frac{1}{2} \sin x
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101. \[
\frac{1}{2} \sin x
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103. \[
\frac{1}{2} \sin x
\]

105. \[
\frac{1}{2} \sin x
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107. \[
\frac{1}{2} \sin x
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109. \[
\frac{1}{2} \sin x
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111. \[
\frac{1}{2} \sin x
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113. \[
\frac{1}{2} \sin x
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115. \[
\frac{1}{2} \sin x
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117. \[
\frac{1}{2} \sin x
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119. \[
\frac{1}{2} \sin x
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121. \[
\frac{1}{2} \sin x
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123. \[
\frac{1}{2} \sin x
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125. \[
\frac{1}{2} \sin x
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127. \[
\frac{1}{2} \sin x
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129. \[
\frac{1}{2} \sin x
\]

131. \[
\frac{1}{2} \sin x
\]

133. \[
\frac{1}{2} \sin x
\]

135. \[
\frac{1}{2} \sin x
\]

137. \[
\frac{1}{2} \sin x
\]

139. \[
\frac{1}{2} \sin x
\]

41. \[
\frac{1}{2} \sin x
\]

43. \[
\frac{1}{2} \sin x
\]

45. \[
\frac{1}{2} \sin x
\]

47. \[
\frac{1}{2} \sin x
\]

49. \[
\frac{1}{2} \sin x
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51. \[
\frac{1}{2} \sin x
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53. \[
\frac{1}{2} \sin x
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55. \[
\frac{1}{2} \sin x
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57. \[
\frac{1}{2} \sin x
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59. \[
\frac{1}{2} \sin x
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61. \[
\frac{1}{2} \sin x
\]

63. \[
\frac{1}{2} \sin x
\]

65. \[
\frac{1}{2} \sin x
\]

67. \[
\frac{1}{2} \sin x
\]

69. \[
\frac{1}{2} \sin x
\]

71. \[
\frac{1}{2} \sin x
\]

73. \[
\frac{1}{2} \sin x
\]

75. \[
\frac{1}{2} \sin x
\]

77. \[
\frac{1}{2} \sin x
\]

79. \[
\frac{1}{2} \sin x
\]

81. \[
\frac{1}{2} \sin x
\]

83. \[
\frac{1}{2} \sin x
\]

85. \[
\frac{1}{2} \sin x
\]

87. \[
\frac{1}{2} \sin x
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89. \[
\frac{1}{2} \sin x
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91. \[
\frac{1}{2} \sin x
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93. \[
\frac{1}{2} \sin x
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95. \[
\frac{1}{2} \sin x
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97. \[
\frac{1}{2} \sin x
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99. \[
\frac{1}{2} \sin x
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101. \[
\frac{1}{2} \sin x
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103. \[
\frac{1}{2} \sin x
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105. \[
\frac{1}{2} \sin x
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107. \[
\frac{1}{2} \sin x
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109. \[
\frac{1}{2} \sin x
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111. \[
\frac{1}{2} \sin x
\]

113. \[
\frac{1}{2} \sin x
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115. \[
\frac{1}{2} \sin x
\]

117. \[
\frac{1}{2} \sin x
\]

119. \[
\frac{1}{2} \sin x
\]

121. \[
\frac{1}{2} \sin x
\]

123. \[
\frac{1}{2} \sin x
\]

125. \[
\frac{1}{2} \sin x
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127. \[
\frac{1}{2} \sin x
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129. \[
\frac{1}{2} \sin x
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131. \[
\frac{1}{2} \sin x
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133. \[
\frac{1}{2} \sin x
\]

135. \[
\frac{1}{2} \sin x
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137. \[
\frac{1}{2} \sin x
\]

139. \[
\frac{1}{2} \sin x
\]

41. \[
\frac{1}{2} \sin x
\]

43. \[
\frac{1}{2} \sin x
\]

45. \[
\frac{1}{2} \sin x
\]

47. \[
\frac{1}{2} \sin x
\]

49. \[
\frac{1}{2} \sin x
\]

51. \[
\frac{1}{2} \sin x
\]

53. \[
\frac{1}{2} \sin x
\]

55. \[
\frac{1}{2} \sin x
\]

57. \[
\frac{1}{2} \sin x
\]

59. \[
\frac{1}{2} \sin x
\]

61. \[
\frac{1}{2} \sin x
\]
Answers to Odd-Numbered Problems  A-37

17. \( \frac{1}{2} x^2 - x + \ln |x + 2| + \ln |x - 1| + C \)
19. \( \frac{1}{2} x^2 - 2 \ln |x| + 7 \ln |x + 2| + 7 \ln |x - 2| + C \)
21. \( \ln x - 3 l + \frac{2}{x - 3} + C \)
23. \( \frac{2}{x + 1} + \frac{2}{x + 1} + \ln |x + 1| + C \)
25. \( 2 \ln x + \ln |x - 4| + \frac{1}{x - 4} + C \)
27. \( -2 \ln x + \frac{2}{\tan^2(\frac{x}{2})} + 2 \ln |x + 4| + C \)
29. \( -3 \ln x - 1 + \frac{1}{3} |x^3| + 4l + C \)
31. \( \frac{1}{125} x^2 - \ln |x - 1| + \frac{1}{25(x - 1)} + \frac{2}{125} x^2 + 4l + C \)
33. \( \sin t - \frac{1}{3 \ln \sin t + 3} - \frac{4}{3} \ln |\sin t| = t| \sin t = 2 \)
35. \( |x| + \frac{2}{x^3 + 4} + C \)
37. \( \frac{\tan x}{2} + 2 \ln |x^2 + 4| + C \)
39. \( \frac{1}{2 \sqrt{1 - x^2}} + \ln |\sin x| + C \)
41. \( \sqrt{2} + \frac{1}{\sqrt{2} - 1} + 6 \sqrt{2} - \frac{1}{6 \sqrt{2}} + C \)

19. \( \sqrt{3} x + \sqrt{3} x + \sqrt{3} x + C \)
21. \( \ln|x + 1| + \sqrt{2} t + 2r - 3l + C \)
23. \( \sqrt{3} y + 3 + C \)
25. \( \sqrt{3} (\sin x - 10) \sqrt{3} x + 5 + C \)
27. \( \frac{1}{2} (1 - \cos x) \sqrt{2} \sin^2 x + C \)
29. \( \sqrt{2} \cos t + \sqrt{2} \cos t - \frac{3}{2} \cos^2 t - 2 \sin t + C \)
31. \( \arctan x \)


1. \( e - 1 \)
3. \( \frac{1}{2} \)
5. \( \frac{1}{2} \)
7. \( e + 2 \ln |x| + C \)
9. \( e + 2 \ln |x| + C \)
11. \( e + 2 \ln |x| + C \)
13. \( e + 2 \ln |x| + C \)
15. \( e + 2 \ln |x| + C \)
17. \( e + 2 \ln |x| + C \)
19. \( e + 2 \ln |x| + C \)
21. \( e + 2 \ln |x| + C \)
23. \( e + 2 \ln |x| + C \)
25. \( e + 2 \ln |x| + C \)
27. \( e + 2 \ln |x| + C \)
29. \( e + 2 \ln |x| + C \)
31. \( e + 2 \ln |x| + C \)
33. \( e + 2 \ln |x| + C \)
35. \( e + 2 \ln |x| + C \)
37. \( e + 2 \ln |x| + C \)
41. \( \ln x - \frac{2}{x} - 2 \left[ \ln \left( x^2 + 3 \right) + \frac{2}{\sqrt{3}} \ln \left( \frac{1}{\sqrt{3}} \right) + C \right] \)

43. (a) \( A \frac{2x+1}{2x+1} + \frac{(2x+1)^2}{(2x+1)^2} = \frac{4x+2}{4x+2} \)

(b) \( A B \frac{x+1}{x+1} + \frac{C}{x+1} + \frac{D}{x+1} = \frac{E}{x+1} \)

(c) \( -A x^2 + B x + C + D \frac{x^2 + x + 10}{x^2 + x + 10} + \frac{E}{x^2 + x + 10} \)

(d) \( A \frac{1-x}{1-x} + \frac{B}{1-x} + \frac{C}{1-x} + \frac{D}{1-x} = \frac{E x + F}{x+1} \)

(e) \( A \frac{1-x}{1-x} + \frac{B}{1-x} + \frac{C}{1-x} + \frac{D}{1-x} = \frac{E x + F}{x+1} \)

(f) \( 2x^2 + x + 10 = \frac{2x^2 + x + 10}{2x^2 + x + 10} + \frac{E x + F}{2x^2 + x + 10} \)

45. \( \ln \left( \frac{2\sqrt{3}}{3} \right) \)

47. \( 2 \pi \ln \frac{\sqrt{3}}{2} \)

49. \( 2x \ln x - 3 - \frac{1}{2} \ln (x^3) \)

51. \( \ln 7 - \frac{1}{2} \)

55. (a) \( \frac{\sin x}{2} \sqrt{\sin^2 x + 4} + 2 \ln \sin x + \sqrt{\sin^2 x + 4} + C \)

(b) \( \frac{\ln \left( 1 + 2x \right)}{2} - \frac{1}{2} \ln \left( 1 - 2x \right) + C \)

57. \( c = \frac{1}{2} \)

Chapter 5 Review and Preview Problems:

1. \( \frac{5}{3} \)

3. \( \frac{1}{3} \)

5. \( \frac{1}{2} \)

7. \( 0 \)

9. \( \infty \)

11. \( \frac{\pi}{2} \)

15. \( \lim_{x \to a} x^2 e^x = 0. \)

17. \( \lim_{x \to a} x^3 e^x = 0. \)

19. \( \lim_{x \to a} x^4 e^x = 0. \)

21. \( a \quad 1 \quad 2 \quad 4 \quad 8 \quad 16 \)

\( 1 - e^{-a} \quad 0.632 \quad 0.865 \quad 0.982 \quad 0.99986 \quad 0.999999887 \)

23. \( a \quad 1 \quad 2 \quad 4 \quad 8 \quad 16 \)

\( \ln(\sqrt{1 + a^2}) \quad 0.2466 \quad 0.8047 \quad 1.4166 \quad 2.0872 \quad 2.7745 \)

25. \( a \quad 2 \quad 4 \quad 8 \quad 16 \)

\( 1 - \frac{1}{a} \quad 0.5 \quad 0.75 \quad 0.875 \quad 0.9375 \)

27. \( a \quad 1 \quad 1/2 \quad 1/4 \quad 1/8 \quad 1/16 \)

\( 4 - 2\sqrt{a} \quad 2 \quad 2.58579 \quad 3 \quad 3.29209 \quad 3.5 \)

1. \( 1 \quad 3 \quad -1 \quad 5 \quad -\frac{1}{3} \quad 7 \quad -\infty \quad 9 \quad 0 \)

11. \( -\frac{1}{3} \quad 13. \quad \frac{1}{2} \quad 15. \quad \frac{1}{2} \quad 17. \quad -\infty \quad 19. \quad \frac{1}{2} \)

21. \( -\infty \quad 23. \quad 1 \)

29. \( a = \frac{1}{2} \)

31. \( \frac{4e^b}{2} \quad \frac{3}{2} \quad 37. \quad 2 \)

39. The ratio of the slopes is 1/2, indicating that the limit of the ratio should be about 1/2.

41. The ratio of the slopes is \(-1/1 = -1\), indicating that the limit of the ratio should be about \(-1\).
1. (a) $C = \beta f(\alpha)$; (b) $\mu = a/\beta$; (c) $\sigma^2 = a/\beta^2$

55. (a) $C = \beta f(\alpha)$; (b) $\mu = a/\beta$; (c) $\sigma^2 = a/\beta^2$

57. (a) $\frac{d}{dx}$; (b) $y$
43. (b) Indefinitely
47. (a) 2; (b) 1
49. (a) \(-1\); (b) \(-\infty\)
51. 1

21. Converges 23. 0.0404 25. 0.1974 27. \(n > 50\)
29. \(n > 5000\)
31. \(n > 50\)
33. \(p > 1\)

39. 272,404.866

13. Converges; Limit Comparison Test
15. Converges; Ratio Test
17. Converges; Limit Comparison Test
19. Converges; Limit Comparison Test
21. Converges; Limit Comparison Test
23. Converges; Ratio Test
25. Converges; Limit Comparison Test
27. Diverges; nth-Term Test
29. Converges; Comparison Test
31. Converges; Ratio Test
33. Converges; Ratio Test
43. (a) Diverges; (b) Converges; (c) Converges; (d) Converges; (e) Diverges
(f) Converges
45. Converges for \(p > 1\), diverges for \(p \leq 1\).

1. \(|S - S_n| \leq 0.065\)
3. \(|S - S_n| \leq 0.417\)
5. \(|S - S_n| \leq 0.230\)
13. Conditionally convergent
15. Divergent
17. Conditionally convergent
19. Absolutely convergent
21. Conditionally convergent
23. Conditionally convergent
25. Absolutely convergent
27. Conditionally convergent
29. Divergent
31. Divergent
33. \(|S - S_n| \leq 0.833\)
45. In 2

1. All \(x\)
3. \(-1 \leq x \leq 1\)
5. \(-1 \leq x \leq 1\)
7. \(-1 \leq x \leq 1\)
9. \(-1 \leq x \leq 1\)
11. All \(x\)
13. \(-1 \leq x \leq 1\)
15. \(-1 \leq x \leq 1\)
17. \(-1 \leq x \leq 1\)
19. \(-2 < x < 2\)
21. All \(x\)
23. \(0 \leq x < 2\)
25. \(-2 < x < 1\)
27. \(-6 \leq x \leq 1\)
29. \(|S - S_n| \neq 0\) then \(\sum a_n^m\) will not converge.

31. \(\sqrt{2}\)
33. \(\frac{1}{x - 2}; \frac{1}{x - 4}\)
35. (a) \(-\frac{1}{2} \leq x \leq \frac{1}{2};\) (b) \(-1 < x \leq \frac{1}{2}\)
37. \(S(x) = \lim_{n \to \infty} \frac{a_0 + a_1 + a_2x^2}{1 - x^2}\)

1. \(-1 + x + x^2 - x^3 + x^4 + \cdots\)
3. \(1 + 3x + 6x^2 + 10x^3 + \cdots\)
5. \(\frac{1}{2} + \frac{3x^2 + 9x^3}{2} + \frac{27x^5}{8} + \cdots\)
7. \(x^2 + x^3 + x^5 + x^6 + \cdots\)
9. \(\frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{12} + \frac{x^5}{20} + \cdots\)
11. \(2x + \frac{2x^3}{3} + \frac{x^5}{5} + \cdots\)
13. \(1 - x + \frac{2x^2}{3} - \frac{x^4}{3} + \frac{x^6}{5} - \cdots\)
15. \(2 + \frac{2x^2}{3} + \frac{2x^4}{5} + \cdots\)
17. \(1 + \frac{x^3}{3} + \frac{3x^4}{8} + \frac{6x^5}{30} + \cdots\)
19. \(x = x^3 + \frac{3x^5}{6} + \frac{3x^7}{40} + \cdots\)
21. \(x^2 + \frac{13x^3}{3} + \frac{29x^4}{15} + \cdots\)
23. \(x^3 + \frac{x^5}{6} + \frac{x^7}{40} + \cdots\)
25. \(\frac{x}{1 + x}\)

27. \(\frac{x}{1 + x}\)
29. (a) \(\frac{x^2}{2} - \frac{x^3}{6} + \cdots;\) (b) \(1 + x + x^3 + \frac{5x^3}{6} + \cdots\)
31. \(x^2 + \frac{7x^3}{8} + \cdots\)
33. \(\frac{x}{1 - x - x^2}\)
35. 3.14159

1. \(x + \frac{x^3}{3} + \frac{x^5}{15} + \cdots;\)
3. \(x + x^3 + \frac{x^5}{3} + \frac{x^7}{35} + \cdots\)
5. \(x - \frac{x^3}{6} + \frac{x^5}{24} + \frac{x^7}{60} + \cdots\)
7. \(1 + x + \frac{x^3}{2} + \frac{x^5}{4} + \cdots\)
9. \(1 + x + \frac{3x^2}{2} + \frac{3x^4}{8} + \cdots\)
11. \(-1 + x + x^4\)
13. \(x^3 - \frac{x^5}{2} + \frac{15x^5}{2} - \frac{120x^7}{6} + \cdots\)
17. \(1 + \frac{x^2}{2} + \frac{x^4}{8} - \frac{16}{120}x^6 + \frac{3x^4}{3} + \frac{x^2}{2} - \frac{8x^6}{120} + \cdots\)
19. \(e^{-(x - 1)^2} + \frac{e^{-(x - 1)^2}}{2(x - 1)^2}\)
21. \(\frac{1}{2} - \frac{\sqrt{2}(x - \pi/2)}{2} - \frac{1}{4}(x - \pi/2)^2 + \frac{\sqrt{2}}{12}(x - \pi/2)^3\)
23. \(3 + 5x(x - 1) + 4(x - 1)^3 + (x - 1)^5\)
27. \(x + 6 + \frac{5x^3}{112} + 29.9045\)
31. \(-1 + x - (x - 1)^2 - (x - 1)^3 + \cdots\)
33. (a) 25. (b) -3. (c) 0. (d) 4e. (e) -4
35. \(x - \frac{x^5}{6} + \frac{x^7}{120} + 5060\)
37. \(-2 + x - x^2 + \frac{5x^3}{6} + \cdots\)
41. \(x - \frac{x^3}{6} + \frac{x^5}{120} + \frac{x^7}{5060}\)
43. \(-2 + x - x^2 + \frac{5x^3}{6} + \cdots\)
45. \(x + \frac{x^2}{2} - \frac{23x^3}{24} - \frac{2x^5}{120} + \cdots\)
47. \(x + x^2 + \frac{x^3}{3} + \frac{x^4}{30} + \cdots\)

A-40 Answers to Odd-Numbered Problems

1. \(1 + 2x + 2x^2 + \frac{5x^3}{3} + \frac{x^4}{3} + \frac{x^5}{5} + \cdots\)
3. \(2x - \frac{1}{3}x^3 + 0.2377\)
5. \(-\frac{1}{3}x^3 + \frac{1}{3}x^4 + 0.1133\)
7. \(x - \frac{1}{3}x^3 + 0.1194\)
9. $e + e(x - 1) + \frac{3}{2}(x - 1)^2 + \frac{4}{3}(x - 1)^3$

11. $\frac{\sqrt{3}}{3} + \frac{4}{3}(x - \pi) + \frac{4\sqrt{3}}{9}(x - \pi)^2 + \frac{8}{9}(x - \pi)^3$

13. $\frac{\pi}{3} - \frac{3}{2}(x - 1) + \frac{3}{4}(x - 1)^2 - \frac{3}{8}(x - 1)^3$

15. $7 + 2(x - 1) + (x - 1)^2 + (x - 1)^3$

17. $f(x) = 1 + x^3 + x^4 + x^5$

(a) 1.1111; (b) 1.9375; (e) 4.0953; (d) 3.1

20. $x^3 + 1$

21. $2\sqrt{\pi}$

33. $\frac{e^6}{3}$

35. $\frac{17}{10}\ln 2$

37. $R_1(x) = \frac{e^x}{2 + e}$

39. $R_2(x) = \frac{\cos(x)}{5040}(x - \pi)^7$

41. $R_1(x) = \frac{(x - 1)^3}{e^x}$

43. $n = 9$

45. $\frac{1}{2} + \frac{x^2}{8} + \frac{x^4}{16} - |R_1(x)| = 0.0276$

47. $1 - \frac{1}{3} + \frac{2}{3}x^2 - \frac{1}{3}x^3; |R_2(x)| < 2.15 \times 10^{-5}$

49. $0.1224; |\text{Error}| < 0.00013025$

51. $n > 42$

53. $A = \frac{1}{2}x^2 - \frac{1}{2}y^2 + \sin x; A = 1 + \frac{x^2}{2}$

55. $|\text{Error}| < 0.00013025$

57. $-1 - (x - 1)^2 + (x - 1)^3 + (x - 1)^4$

59. $0.681998; |R_1| < 6.19 \times 10^{-5}$
1. (a) $y = x - 1$ (b) $y = -x + 3$
3. (2, -2.4), (-2, 2.4), (2.4, -2.4), (-2.4, -2.4)
5. $y = 2V3x + 4$
7. (5, 4V3), (-5, 4V3), (5, -4V3), (-5, -4V3);
   $T_1: 4V3x + 15y = 80V3; T_2: 5V3x - 4y = 9V3; a = 90^\circ$
9. $r = 5; \theta = \sin^{-1}(0.6)$
11. $x = y = 4V2$

1. Focus at (1,0);
   directrix $x = -1$
3. Focus at (0, -3);
   directrix $y = 3$
5. Focus at (1/2, 0);
   directrix $x = -1/2$
7. Focus at (0, 1/2);
   directrix $y = -1/2$
9. $y^2 = 8x$
11. $x^2 = -8y$
13. $y^2 = -16x$
15. $y^2 = 1/4x$
17. $x^2 = -20y$
19. $y = -2x - 2;
    y = 1/2x - 3/2$
21. $y = 4x - 8;
    y = -1/4x + 9$

23. $y = \frac{\sqrt{5}}{2}x - \frac{3\sqrt{5}}{2}$
25. $y = -\sqrt{2}x + 3;
    y = -\frac{\sqrt{5}}{2}x - \frac{21\sqrt{5}}{5}$

27. $(4, 2V3)$

29. $y = \frac{1}{3}x - 3$
35. 14.8 million mi
37. $2p$
39. $l = 4p$
41. $y = \frac{dx^3}{3H}$

1. Horizontal ellipse 3. Vertical hyperbola
5. Vertical parabola (opens up) 7. Vertical ellipse

9.
11.
13.
15.
17. $\frac{x^2}{36} + \frac{y^2}{27} = 1$
19. $\frac{x^2}{200} + \frac{y^2}{225} = 1$
21. $\frac{x^2}{25} + \frac{y^2}{32} = 1$
23. \( \frac{y}{16} = \frac{x^2}{9} = 1 \)
25. \( \frac{x^2}{64} - \frac{y^2}{16} = 1 \)
27. \( \frac{x^2}{16} + \frac{y^2}{12} = 1 \)
29. \( \frac{x^2}{9} - \frac{y^2}{20} = 1 \)
31. \( \frac{x^2}{98} + \frac{y^2}{169} = 1 \)
33. \( \frac{x^2}{36} - \frac{y^2}{13} = 1 \)
35. \( x + \sqrt{6}y = 9 \)
37. \( x - \sqrt{6}y = 9 \)
39. \( 5x + 12y = 169 \)
41. \( y = 13 \)
43. 8.66 ft
45. \( \frac{2k^2}{a} \)
47. 0.58 AU
49. 0.05175
51. \( (\sqrt{3}, \frac{1}{2}), (\sqrt{3}, \frac{3}{2}) \)
53. \((7, 3), (3, -7)\)
55. \( mb \)
57. \( \frac{ab}{3a^2} \left[ (a^2 + b^2)^{3/2} - 3a^2 \sqrt{a^2 + b^2} + 2a^2 \right] \)
59. \( a\sqrt{2} \) by \( b\sqrt{2} \)
61. \((6, 5\sqrt{3}) \)

63. Elliptical mirror

Other focus of ellipse

Common focus of parabola and ellipse

69. \((\sqrt{3}, 5) \)
73. \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \)


15.

17.

19.

21.
53. (a) \( y = x^2 - x \); (b) \( x = \sqrt{\frac{y}{2}} - y \);
(c) \( (x - \frac{1}{2})^2 + (y - \frac{3}{2})^2 = \frac{5}{4} \)

55. If \( K < -1 \), the conic is a vertical ellipse. If \( K = -1 \), the conic is a circle. If \(-1 < K < 0 \), the conic is a horizontal ellipse. If \( K = 0 \), the conic is a horizontal parabola. If \( K > 0 \), the conic is a horizontal hyperbola.

59. \( u = x \cos \theta + y \sin \theta, v = -x \sin \theta + y \cos \theta \)

61. \( \left( -\frac{1}{2}, -\frac{1}{2} \right), \left( \frac{1}{2}, \frac{1}{2} \right) \)

67. (a) \( -2 < B < 2 \); (b) \( B = 0 \); (c) \( B < -2 \) or \( B > 2 \);
(d) \( B = \pm 2 \)

13. (a) Simple; closed
(b) \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \)

15. (a) Not simple; closed
(b) \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \)

17. (a) Not simple; closed
(b) \( x + y = 9 \)

19. (a) Not simple; not closed
(b) \( y = -8x^2(1 - x^2) \)

21. \( \frac{dy}{dx} = 2r \); \( \frac{d^2y}{dx^2} = \frac{1}{3}r \)

23. \( \frac{dy}{dx} = \frac{3\sqrt{5}}{4}r \); \( \frac{d^2y}{dx^2} = \frac{3\sqrt{5}}{16}r \)

25. \( \frac{dy}{dx} = \cos \theta \); \( \frac{d^2y}{dx^2} = -\cos^2 \theta \)

29. \( \frac{dy}{dx} = \frac{(1 - 2l)(1 + l^2)}{2l(1 + r^2)} \); \( \frac{d^2y}{dx^2} = \frac{3r^4 + 5r^2 - 6r + 10r^2 - 9r + 3(1 + r^2)^3}{6r(1 + r^2)^4} \)}
31. \( y - 8 = 3(x - 4) \)  
33. \( y + \frac{2}{\sqrt{3}} = -2\left(x - \frac{4}{\sqrt{3}}\right) \)

35. \( 3\sqrt{3} \)  
37. \( \frac{1}{3}(3\sqrt{3} - 8) \)  
39. \( 16\sqrt{2} - 8 \)  
41. \( \frac{\sqrt{3}\sqrt{3} - 227\sqrt{227}}{243} \)  
43. \( \frac{3\sqrt{2}}{25} \)  
45. \( \frac{1}{2}\ln 2 \)

47. (a) \( 2\pi \)  
(b) \( 6\pi \)  
(c) The curve in part (a) goes around the unit circle once.  
The curve in part (b) goes around the unit circle three times.

49. \( 4\pi^2 \)  
51. \( 4\pi^2 \)  
53. \( \frac{\sqrt{2}}{2}(9\sqrt{3} - 1) \)  
55. \( -\frac{1}{2} \)

57. \( 0 \)

59. \( \frac{1}{10^4} \)

61. \( x = (a - b)\cos t + b\cos\left(\frac{a - b}{b}t\right) \)  
\( y = (a - b)\sin t - b\sin\left(\frac{a - b}{a}t\right) \)

65. \( L = \frac{16\pi}{3} \)

67. (a)

(b)

(c)
Answers to Odd-Numbered Problems

75. Quadrant I for $t > 0$, quadrant II for $-1 < t < 0$, quadrant III for no $t$, quadrant IV for $t < -1$.

11. $r = \frac{2}{3\sin \theta - \cos \theta}$

13. $r = -2 \csc \theta$

15. $r = 2$

17. $x = 0$

19. $x = -3$

21. $y = 1$

23. Circle

25. Line

27. Circle

29. Parabola; $e = 1$

31. Ellipse; $e = \frac{1}{2}$

33. Parabola; $e = 1$
35. Hyperbola, $e = 2$

39. $2\sqrt{e}$

41. 0.83

43. 25 million mi

45. $e = 0.1$

$e = 0.5$

$e = 0.9$

$e = 1$

$e = 1.1$

$e = 1.3$
21. \[\text{Graph 1}\]

23. \[\text{Graph 2}\]

47. \[\text{Graph 4}\]

51. (a) The graph for \(\phi = 0\) is the graph for \(\phi \neq 0\) rotated by \(\phi\) counterclockwise about the pole.

(b) As \(n\) increases, the number of “leaves” increases.

53. The spiral will unwind clockwise for \(c < 0\). The spiral will unwind counterclockwise for \(c > 0\).

55. (a) III; (b) IV; (c) I; (d) II; (e) VI; (f) V

57. \([6, \frac{\pi}{3}]; (6, \frac{2\pi}{3})\]

35. \((0, 0), \left(\frac{3\sqrt{2}}{2}, \frac{\pi}{3}\right)\)

59. \([\frac{\pi}{2}, \pi]\)

41. \[\pi a^2\]

33. \[\left(\frac{3\sqrt{2}}{2}, \frac{\pi}{3}\right)\]

51. \[\left(\frac{\pi}{2}, \frac{\pi}{3}\right)\]

43. (a) \(r = \frac{45}{\cos \theta}\); (b) \(r = 6\); (c) \(r = \frac{1}{\sqrt{\cos 2\theta}}\)

(d) \(r = \frac{1}{\sqrt{2 \sin 2\theta}}\); (e) \(r = \frac{2}{\sin \theta - 3 \cos \theta}\)

(f) \(r = -2 \sin \theta \pm \sqrt{\sin^2 \theta + 6 \cos^2 \theta}\)

(g) \(r = -\cos \theta + 2 \sin \theta \pm \sqrt{(\cos \theta - 2 \sin \theta)^2 + 25}\).

45. (a) VII; (b) I; (c) VIII; (d) IV; (e) V; (f) II; (g) VI; (h) IV
13. $\frac{3}{2} \pi - \frac{1}{2} \cos^{-1}\frac{1}{3} + 3\sqrt{3}$

15. $5\pi$

17. $5\pi$

19. $4\sqrt{3} - \frac{1}{3}\pi$

21. $9\sqrt{2} - \frac{22}{5}$

23. (a) $\frac{1}{\sqrt{2}}$  
(b) $-1$  
(c) $\frac{\sqrt{2}}{2}$  
(d) $-\frac{7}{\sqrt{3}}$

25. $(-1, \frac{3}{4})$, $(\frac{3}{4}, \sin^{-1}\frac{1}{4})$, $(\frac{1}{4}, \pi - \sin^{-1}\frac{1}{4})$

27. $8a$

29. $\sqrt{n}$ if $n$ is even, $2\sqrt{n}$ if $n$ is odd.

31. (a) $a^2 \tan^{-1}\frac{1}{2} + b\left(\frac{x}{2} - \tan^{-1}\frac{1}{2}\right) = ab$

33. $x^2 - (k^2 - 1)x + (2 - k^2)\cos\left(\frac{\pi}{2}\right) - k\sqrt{4 - k^2}$

35. 1.26a

37. $4\pi, 26.73$

39. 63.46

1. False  
3. False  
5. True  
7. True  
9. False

11. True  
13. False  
15. True  
17. True  
19. False

21. False  
23. False  
25. False  
27. False  
29. True

31. False  
33. True

1. (a) (5); (b) (9); (c) (4); (d) (3); (e) (2); (f) (6); (g) (5); (h) (1); (i) (7); (j) (6)

3. Ellipse  
Focus at $(0, \pm \sqrt{3})$  
Vertices at $(0, \pm 3)$

5. Parabola  
Focus at $(0, -\frac{3}{4})$  
Vertex at $(0, 0)$

7. Ellipse  
Focus at $(\pm 4, 0)$  
Vertices at $(\pm 5, 0)$

9. Parabola  
Focus at $(0, 0)$  
Vertex at $(0, \frac{1}{4})$

11. $\frac{x^2}{16} + \frac{y^2}{12} = 1$

13. $9x = -2y$

15. $x^2 - y^2 = 1$

17. $(x - 1)^2 + (y - 2)^2 = 1$

19. Circle  
21. Parabola

23. $r = \frac{1}{2}; x = -\frac{1}{2}.$ hyperbola; $4\sqrt{6}$

25. $y = \frac{1}{4}(x - 2)$

27. $(x + 2)^2 + (y - 1)^2 = 1$

29. $y = -\frac{1}{2}(x - 7)$  
31. $27\sqrt{2}$
A-50 Answers to Odd-Numbered Problems

47. \(-1\)  49. \(\pi\)  51. \(\frac{\pi}{2}\)

53. (a) I; (b) IV; (c) III; (d) II; (e) V

7. \(x = h \cos \theta\)
   \(y = h \sin \theta\)

9. \(\frac{1}{24} \left( 328 \cdot 3^2 - 8 \right) = 24.4129\)

11. \(\pi |a|\)  13. Between \((0.8, 2.6)\), distance is \(\sqrt{0.8}\)

15. (a) \(v(t) = 2t - 6; a(t) = 2\)  (b) \(t > 3\)
1. \( A(1, 2, 3), B(2, 0, 1), C(-2, 4, 5), D(0, 3, 0), E(-1, -2, -3) \)

3. \( x = 0; y = 0; z = 0 \)

5. (a) \( \sqrt{13} \); (b) 5; (c) \( \sqrt{(x + 3)^2 + (y + 4)^2 + z^2} = 3 \)

11. (a) \( (x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 25 \);
(b) \( (x + 2)^2 + (y + 3)^2 + (z + 6)^2 = 5 \);
(c) \( (x - \pi)^2 + (y - e)^2 + (z - \sqrt{2})^2 = \pi \)

13. \( (0, -7, 4), 10 \)

15. \( \left[ \frac{1}{2}, -1, -2 \right] \)

17. \( \sqrt{\frac{2}{5}} \)

21. \( \sqrt{2} \)

23. \( \frac{2}{5} \sqrt{2} \)

25. \( 2 \sqrt{6} \)

27. \( 16, 59 \)

29. \( 2 \sqrt{13} \)

31. \( 2 \pi \sqrt{13} \)

33. \( 7.2273 \)

35. \( 34.8394 \)

37. \( (x - 1)^2 + (y - 1)^2 + (z - \frac{1}{2})^2 = 9 \)

39. \( (x - 6)^2 + (y - 6)^2 + (z - 6)^2 = 56 \)

41. (a) Plane parallel to and 2 units above the xy-plane;
(b) Plane perpendicular to the xy-plane, whose trace in the xy-plane is the line \( x = y \);
(c) Union of the \( xy \)-plane \((x = 0)\) and the \( yz \)-plane \((y = 0)\);
(d) Union of the three coordinate planes;
(e) Cylinder of radius 2, parallel to the \( z \)-axis;
(f) Top half of the sphere with center \((0, 0, 0)\) and radius 3

43. Center \((1, 2, 5)\), radius 4

45. \( \frac{15 \pi}{12} \)
Answers to Odd-Numbered Problems

5. (a) 1; (b) 4; (c) \( \frac{\sqrt{5}}{5} \); (d) -2

6. (a) \( \theta_a = 90^\circ \); \( \theta_b = 90^\circ \); \( \theta_c = 125.26^\circ \)
   (b) \( a_1 = 45^\circ \); \( \beta_2 = 45^\circ \); \( \gamma_2 = 90^\circ \)
   (c) \( \alpha = 131.81^\circ \); \( \beta = 48.19^\circ \); \( \gamma = 70.53^\circ \)

7. (a) \( \frac{10}{\sqrt{93}} \) + \( \frac{40}{\sqrt{93}} \) = \( \frac{50}{\sqrt{93}} \)
   (b) \( \frac{10}{\sqrt{93}} \) - \( \frac{40}{\sqrt{93}} \) = \( \frac{-30}{\sqrt{93}} \)

8. \( \cos^{-1} \left( \frac{11}{\sqrt{129}} \right) \)

9. \( 2 \sqrt{3} \)

11. (a) \{t \in \mathbb{R}: t \leq 3\}; (b) \{t \in \mathbb{R}: t < 20, \text{ if } t \text{ not an integer}\}
   (c) \{t \in \mathbb{R}: -3 \leq t \leq 5\}

12. (a) \( y(3t + 4)^2 + 2(3t + 4)^2 + 2(2t^2 + 1) \text{ if } t > 1 \)
   (b) \( \sin(2t) - 3 \sin(2t) + 2k \)
   (c) \( 2 \cos(2t) - 9 \cos(3t) + 2k \)

13. \( -2r^2 + \frac{4}{r} + \frac{4}{r} \ln(r) \)

14. \( -\left( \frac{5}{2} \right)^{1/2} \) + \( \frac{7}{2} \sqrt{1 - \frac{1}{3}} \)

15. \( v(1) = 4i + 10j + 2k; a(1) = 10j; s(1) = 2\sqrt{10} \)

16. \( v(2) = -\frac{i}{4} - \frac{j}{4} + 10k; a(2) = \frac{i}{4} + \frac{j}{4} + 100k \)

17. \( s(2) = \frac{6.286737}{36} \)

18. \( v(2) = 4j + \frac{20}{9}k; a(2) = 4j - \frac{1}{9} \sqrt{2} k \)

19. \( v(\pi) = -j + k; a(\pi) = 1, s(\pi) = \sqrt{2} \)

20. \( v(t) = 2i + j - c^2 k; a(t) = 2r - 2r^2 + c^2 k \)

21. \( s(2) = \sqrt{4 + 400} \)

22. \( 2\sqrt{v} \)

23. \( 34 \)

24. \( \sqrt{4 + 9\pi^2} \)

25. \( (c) + 2(2\pi + 1) \text{ if } \left( \frac{2\pi}{3} \right) \}

26. \( s(t) = \left\{ t \in \mathbb{R} : -3 \leq t \leq 2 \right\} \)
21. (b) \(2x + y - z = 7\); (c) \((-1, 2, -1)\); (d) \(\sqrt{6}\)

23. \(\frac{\sqrt{3}}{3} \quad \frac{y - 3\sqrt{3}}{\sqrt{3}} \quad \frac{z - \frac{3}{2}}{\frac{3}{2}}\)

25. \(3x - 4y + 5z = -22\)

27. \(\left(-\frac{3}{2}, 0 , \frac{37}{4\sqrt{7}}\right)\)

29. \(\left(-\frac{3}{2}, 1, 0\right)\)

31. (a) \(\frac{8\sqrt{2}}{3}\); (b) \(\frac{3\sqrt{26}}{7}\)

1. \(v(1) = (1, 2); \quad \mathbf{a}(1) = (0, 2)\)
\[T(1) = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right); \quad \kappa = \frac{2}{\sqrt{50}}\]

3. \(v(\pi) = (1, 0, -\pi); \quad \mathbf{a}(\pi) = (0, 2, 0)\)
\[T(\pi) = \left(\frac{1}{\sqrt{5}}, 0 , \frac{2}{\sqrt{5}}\right); \quad \kappa = \frac{2}{5}\]

5. \(v(\pi) = \left(\frac{\pi}{4}, 0, -\pi\right); \quad \mathbf{a}(\pi) = \left(\frac{\pi}{2}, 5, 0\right)\)
\[T(\pi) = \left(\frac{\pi}{\sqrt{400 + \pi^2}}, \frac{20}{400 + \pi^2}\right); \quad \kappa \approx 0.195422\]

7. \(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\)

9. \(3 + \frac{4}{\sqrt{2}} + \frac{24\sqrt{2}}{125}\)

11. \(\frac{2}{\sqrt{13}} - \frac{3}{\sqrt{13}} + \frac{6}{13\sqrt{13}}\)

13. \(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\)

15. \(\kappa = \frac{4}{17\sqrt{17}}; \quad R = \frac{17\sqrt{17}}{4}\)

17. \(\kappa = \frac{2}{3\sqrt{3}}; \quad R = \frac{3\sqrt{3}}{2}\)

19. \(\kappa = \frac{3}{2}; \quad R = \frac{3}{2}\)

21. \(\kappa = \frac{3}{2}; \quad R = \frac{3}{2}\)

23. \(\kappa = \frac{4}{5\sqrt{5}}; \quad R = \frac{5\sqrt{5}}{4}\)

25. \(\kappa = \frac{3}{5\sqrt{10}}; \quad R = \frac{5\sqrt{10}}{3}\)

27. \(\sqrt{11}; \quad T = \left(-\frac{2}{\sqrt{21}}, -\frac{1}{\sqrt{21}}, \frac{4}{21}\right)\)
\[N = \left(-\frac{5}{\sqrt{77}}, -\frac{6}{\sqrt{77}}, -\frac{1}{\sqrt{77}}\right)\]
\[B = \left(-\frac{4}{\sqrt{33}}, -\frac{4}{\sqrt{33}}, 0\right)\]

29. \(\kappa = \frac{3}{2}; \quad T = \left(-\frac{3}{\sqrt{13}}, 0, \frac{2}{\sqrt{13}}\right); \quad N = (0, 1, 0);\]
\[B = \left(-\frac{2}{\sqrt{13}}, 0, \frac{3}{\sqrt{13}}\right)\]

31. \(\kappa = \left(\text{sech}^2\right)^{\frac{1}{2}}; \quad T = \text{tanh}^2 + \text{sech}^2\); \quad N = \text{sech}^2 \quad - \text{tanh}^2 4; \quad B = -k\)

33. \(\kappa = \left(\text{sech}^2\right)^{\frac{1}{2}}; \quad T = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}; \quad N = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}; \quad B = -k\)

35. \(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}; \quad R = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\)

37. \(0, 0, 0\)

39. \(\left(-\frac{2}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\)

41. \(a_T = \frac{12}{\sqrt{13}}; \quad \mathbf{a}_T = \frac{18}{\sqrt{13}}\)

43. \(a_T = -\sqrt{2} a_T - \sqrt{2}\)
A-54 Answers to Odd-Numbered Problems

45. \( a_T = \frac{d(\mathbf{a}_T)}{dt} \quad a_T = \frac{3\mathbf{e}}{\sqrt{41}} \)

47. \( a_V(1) = \frac{d(\mathbf{a}_V)}{dt} \quad a_V(1) = 2(\frac{\mathbf{e}}{\sqrt{3}}) \)

49. \( a_N(0) = 0; \quad a_N(0) = \mathbf{e} \)

51. \( a_T(3) = \frac{\mathbf{e}}{\sqrt{55}}; \quad a_N(3) = 6(\frac{\mathbf{e}}{\sqrt{55}}) \)

53. \( (0,0); (1,0), (-1,0) \)

55. The speed is constant; the curvature is zero.

57. \( \cos(5j) - (\sin 5j) \mathbf{i} + 7k \)

59. \( 5T + 5N; -4 = 7 \)

61. \( 72 \text{ ft/s} \)

67. \( P(x) = 10x^3 - 15x^4 + 6x^2 \)

71. \( \frac{3}{8} \)

73. \( \text{max} = 0.7606; \text{min} = 0.1248 \)

85. \( (6,0,8); \sqrt{5x^2 + 1} \)

1. Elliptic cylinder

3. Plane

5. Circular cylinder

7. Ellipsoid

9. Elliptic paraboloid

11. Cylinder

13. Hyperbolic paraboloid

15. Elliptic paraboloid

17. Plane

19. Hemisphere

21. (a) Replacing \( x \) by \( -x \) results in an equivalent equation.

(b) Replacing \( x \) by \( -x \) and \( y \) by \( -y \) results in an equivalent equation.

(c) Replacing \( x \) by \( -x \), \( y \) by \( -y \), and \( z \) by \( -z \) results in an equivalent equation.

23. All central ellipsoids are symmetric with respect to

(a) the origin, (b) the \( z \)-axis, and (c) the \( xy \)-plane.

25. All central hyperboloids of two sheets are symmetric with respect to

(a) the origin, (b) the \( z \)-axis, and (c) the \( yz \)-plane.

27. \( y = 2x^2 + 2z^2 \)

29. \( 4x^2 + 3y^2 + 4z^2 = 12 \)

31. \( (0, \pm 2\sqrt{5}, 4) \)

33. \( \frac{\max(2 - h^2)}{c^2} \)

35. Major diameter 4; minor diameter \( 2\sqrt{2} \)

37. \( x^2 + y^2 = 9z^2 = 0 \)

1. Cylindrical to Spherical: \( \rho = \sqrt{r^2 + z^2}; \quad \cos \phi = \frac{z}{\sqrt{r^2 + z^2}} \)

2. Spherical to Cylindrical: \( r = \rho \sin \phi \); \( z = \rho \cos \phi \); \( \theta = \theta \)

3. (a) \( (3\sqrt{3}, 3, -2) \);

(b) \( (-2, -2\sqrt{3}, -8) \)

5. (a) \( (\sqrt{3}, 3, -\frac{\pi}{6}) \);

(b) \( (3, \frac{\pi}{4}, \frac{\pi}{3}) \)
45. \( a_T = \frac{4 \pi a}{3 \sqrt{41}} \) 
47. \( a_T(1) = \frac{4}{\sqrt{14}} \); \( a_N(1) = \frac{2\sqrt{7}}{7} \) 
49. \( a_T(0) = 0; \ a_N(0) = \frac{\sqrt{2}}{1} \) 
51. \( a_T(3) = 36. \ a_N(3) = 6\sqrt{\frac{2}{55}} \) 
53. \((0,0); (1,0), (-1,0, 0)\)

55. The speed is constant; the curvature is zero.
57. \( (\cos 5) = (\sin 5) + 7 \) 
61. 72 ft/s 
67. \( P(x) = 10x^3 - 15x^4 + 6x^5 \) 
73. \( \frac{1}{8\sqrt{2}} \) 
75. \( \sqrt{0.7066}; \ \text{min} = 0.1248 \)

85. \((6, 0, 8); \sqrt{0.7066} + 1 \)

1. Elliptic cylinder 
2. Plane
3. Plane
4. Ellipsoidal
5. Elliptical cylinder
6. Elliptical paraboloid
7. Elliptic paraboloid

1. Elliptical to Cylindrical: \( p = \sqrt{r^2 + z^2}, \ \phi = \phi \) 
2. Spherical to Cylindrical: \( \rho = \rho \sin \phi, \ \phi = \rho \cos \phi, \ \theta = \theta \) 
3. (a) \( \{3\sqrt{3}, 3, -2\} \); (b) \( \{-2, -2\sqrt{3}, -8\} \) 
5. (a) \( \{4\sqrt{2}, \frac{3\pi}{2}, \frac{\pi}{2}\} \); (b) \( \{4, \frac{5\pi}{2}, \frac{\pi}{2}\} \)
13. maximum value of \( f \) on \([0, 4]\) is 5; minimum value is -15

15. \( S(t) = 2\pi t^2 + \frac{16}{t} \)

1. (a) 5; (b) 0; (c) 6; (d) \( a^2 + a^2 \); (e) \( 2a^2 \);
   (f) \( (2, 4) \) is not in the domain of \( f \). Domain is set of all \((x, y)\) such that \( y > 0 \).

3. (a) 0; (b) 2; (c) 16; (d) -4.2469;

5. 7

11.

13.

15.

17.

19.

21.

23.

25. (a) San Francisco (b) northwest; southeast
   (c) southwest or northeast

27. The set of all points on and outside the sphere \( x^2 + y^2 + z^2 = 16 \).

29. The set of all points on and inside the ellipsoid \( x^2/9 + y^2/16 + z^2/1 = 1 \).

31. All points in \( \mathbb{R}^3 \) except the origin (0, 0, 0).

33. The set of all spheres with centers at the origin.

35. A set of hyperboloids of revolution about the z-axis when \( k = 0 \). When \( k \neq 0 \), the level surface is an elliptic cone.

37. A set of hyperbolic cylinders parallel to the z-axis when \( k \neq 0 \). When \( k = 0 \), the level surface is a pair of planes.

39. (a) All points in \( \mathbb{R}^4 \) except the origin (0, 0, 0, 0).
   (b) All points in \( \mathbb{R}^n \).
   (c) All points in \( \mathbb{R}^n \) that satisfy \( x_1^2 + x_2^2 + \cdots + x_n^2 \leq 1 \).

41. (a) gentle climb; steep climb; (b) 6490 ft, 3060 ft

43.

45.
1. \( f_y(x,y) = 8(2x - y)^3; \) \( f_y(x,y) = -4(2x - y)^3 \)
2. \( f_y(x,y) = (x^2 + y^2)/(x^2y); \) \( f_y(x,y) = -(x^2 + y^2)/(x^2y) \)
3. \( f_y(x,y) = e^x \cos y; \) \( f_y(x,y) = e^x \sin y \)
4. \( f_y(x,y) = x^2 - y^2; \) \( f_y(x,y) = -y(x^2 - y^2)^{1/2} \)
5. \( f_y(x,y) = x^2 \cos y; \) \( f_y(x,y) = e^x \sin y \)
6. \( f_y(x,y) = 4/[1 + (4x - 7y)^2]; \) \( f_y(x,y) = -7/[1 + (4x - 7y)^2] \)
7. \( f_y(x,y) = y(x^2 - y^2)^{1/2}; \) \( f_y(x,y) = -y(x^2 - y^2)^{1/2} \)
8. \( f_y(x,y) = -2xy \sin (x^2 + y^2); \) \( f_y(x,y) = -2xy \sin (x^2 + y^2) \)
9. \( f_y(x,y) = -2x^2 \sin (x^2 + y^2) + \cos (x^2 + y^2) \)
10. \( f_y(x,y) = 2 \cos x \cos y; \) \( f_y(x,y) = -2 \sin x \sin y \)
11. \( f_y(x,y) = 12x^2 - 15x^3 y = f_y(x,y) \)
12. \( F_y(3, -2) = \frac{1}{3} F_y(3, -2) = \frac{-1}{3} \)
13. \( f_y(\sqrt{3}, -2) = -\frac{1}{\sqrt{3}} f_y(\sqrt{3}, -2) = -\frac{1}{\sqrt{3}} \)
14. \( \sum (x^2 y^3 - 12x^2) \)
15. \( (a) (\partial f/\partial y); \) \( (b) (\partial f/\partial y) \)
16. \( (c) (\partial f/\partial y) \)
17. \( (d) f_y(x,y) = -6x^2 \sin y \)
18. \( f_y(x,y) = f_y(x,y) \)
19. \( F(x,y) = \frac{1}{6} F(x,y) \)
20. \( F(x,y) = \frac{1}{6} F(x,y) \)
21. \( f(x,y) = \frac{1}{6} f(x,y) \)
22. \( f(x,y) = \frac{1}{6} f(x,y) \)
23. \( f(x,y) = \frac{1}{6} f(x,y) \)
24. \( f(x,y) = \frac{1}{6} f(x,y) \)
25. \( f(x,y) = \frac{1}{6} f(x,y) \)
26. \( (a) (x^2 + y^2)^{1/2}; \) \( (b) (x^2 + y^2)^{1/2} \)
27. \( (x^2 + y^2)^{1/2}; \) \( (x^2 + y^2)^{1/2} \)
28. \( (x^2 + y^2)^{1/2}; \) \( (x^2 + y^2)^{1/2} \)
29. \( (x^2 + y^2)^{1/2}; \) \( (x^2 + y^2)^{1/2} \)
30. \( (x^2 + y^2)^{1/2}; \) \( (x^2 + y^2)^{1/2} \)
31. \( (x^2 + y^2)^{1/2}; \) \( (x^2 + y^2)^{1/2} \)
32. \( (x^2 + y^2)^{1/2}; \) \( (x^2 + y^2)^{1/2} \)
33. \( (x^2 + y^2)^{1/2}; \) \( (x^2 + y^2)^{1/2} \)
34. \( (x^2 + y^2)^{1/2}; \) \( (x^2 + y^2)^{1/2} \)
35. \( (x^2 + y^2)^{1/2}; \) \( (x^2 + y^2)^{1/2} \)
36. \( (x^2 + y^2)^{1/2}; \) \( (x^2 + y^2)^{1/2} \)
37. \( (a) \phi'(\partial f/\partial y); \) \( (b) \phi'(\partial f/\partial y) \)
38. \( (c) \phi'(\partial f/\partial y) \)
39. \( (d) \phi'(\partial f/\partial y) \)
40. \( (e) \phi'(\partial f/\partial y) \)
41. \( (f) \phi'(\partial f/\partial y) \)
42. \( (g) \phi'(\partial f/\partial y) \)
43. \( (h) \phi'(\partial f/\partial y) \)
44. \( (i) \phi'(\partial f/\partial y) \)
45. \( (j) \phi'(\partial f/\partial y) \)
46. \( (k) \phi'(\partial f/\partial y) \)
47. \( (l) \phi'(\partial f/\partial y) \)
48. \( (m) \phi'(\partial f/\partial y) \)
49. \( (n) \phi'(\partial f/\partial y) \)
50. \( (o) \phi'(\partial f/\partial y) \)
51. \( (p) \phi'(\partial f/\partial y) \)
52. \( (q) \phi'(\partial f/\partial y) \)
53. \( (r) \phi'(\partial f/\partial y) \)
54. \( (s) \phi'(\partial f/\partial y) \)
55. \( (t) \phi'(\partial f/\partial y) \)
56. \( (u) \phi'(\partial f/\partial y) \)
57. \( (v) \phi'(\partial f/\partial y) \)
58. \( (w) \phi'(\partial f/\partial y) \)
59. \( (x) \phi'(\partial f/\partial y) \)
60. \( (y) \phi'(\partial f/\partial y) \)
61. \( (z) \phi'(\partial f/\partial y) \)
29. (a) The gradient points in the direction of greatest increase of the function.
(b) No.

1. \( \frac{\partial f}{\partial x} = \frac{3}{2} 
\frac{\partial f}{\partial y} = \frac{1}{2} 
\frac{\partial f}{\partial z} = \frac{1}{2} 
\)

3. \( \frac{\partial g}{\partial x} = \frac{1}{2} 
\frac{\partial g}{\partial y} = -1 
\frac{\partial g}{\partial z} = 1 
\)

5. \( \frac{\partial h}{\partial x} = 2 
\frac{\partial h}{\partial y} = 0 
\frac{\partial h}{\partial z} = 0 
\)

7. \( \frac{\partial i}{\partial x} = 1 
\frac{\partial i}{\partial y} = 0 
\frac{\partial i}{\partial z} = 0 
\)

9. \( \frac{\partial k}{\partial x} = 0 
\frac{\partial k}{\partial y} = 1 
\frac{\partial k}{\partial z} = 0 
\)

11. \( \frac{\partial l}{\partial x} = -1 
\frac{\partial l}{\partial y} = 0 
\frac{\partial l}{\partial z} = 0 
\)

13. \( \frac{\partial m}{\partial x} = 0 
\frac{\partial m}{\partial y} = 1 
\frac{\partial m}{\partial z} = 0 
\)

15. \( \frac{\partial n}{\partial x} = 1 
\frac{\partial n}{\partial y} = 0 
\frac{\partial n}{\partial z} = 0 
\)

17. \( \frac{\partial o}{\partial x} = 0 
\frac{\partial o}{\partial y} = 1 
\frac{\partial o}{\partial z} = 0 
\)

19. \( \frac{\partial p}{\partial x} = 0 
\frac{\partial p}{\partial y} = 1 
\frac{\partial p}{\partial z} = 0 
\)

21. \( \frac{\partial q}{\partial x} = 0 
\frac{\partial q}{\partial y} = 0 
\frac{\partial q}{\partial z} = 1 
\)

23. \( \frac{\partial r}{\partial x} = 1 
\frac{\partial r}{\partial y} = 0 
\frac{\partial r}{\partial z} = 0 
\)

25. \( \frac{\partial s}{\partial x} = 0 
\frac{\partial s}{\partial y} = 1 
\frac{\partial s}{\partial z} = 0 
\)

27. \( \frac{\partial t}{\partial x} = 0 
\frac{\partial t}{\partial y} = 0 
\frac{\partial t}{\partial z} = 1 
\)

29. \( \frac{\partial u}{\partial x} = 1 
\frac{\partial u}{\partial y} = 0 
\frac{\partial u}{\partial z} = 0 
\)

31. \( \frac{\partial v}{\partial x} = 0 
\frac{\partial v}{\partial y} = 1 
\frac{\partial v}{\partial z} = 0 
\)

33. \( \frac{\partial w}{\partial x} = 0 
\frac{\partial w}{\partial y} = 0 
\frac{\partial w}{\partial z} = 1 
\)

35. \( \frac{\partial x}{\partial y} = 0 
\frac{\partial x}{\partial z} = 0 
\frac{\partial x}{\partial w} = 1 
\)

37. \( \frac{\partial y}{\partial x} = 0 
\frac{\partial y}{\partial y} = 1 
\frac{\partial y}{\partial z} = 0 
\)

39. \( \frac{\partial z}{\partial x} = 0 
\frac{\partial z}{\partial y} = 0 
\frac{\partial z}{\partial w} = 1 
\)

41. \( \frac{\partial u}{\partial x} = 1 
\frac{\partial u}{\partial y} = 0 
\frac{\partial u}{\partial z} = 0 
\)

43. \( \frac{\partial v}{\partial x} = 0 
\frac{\partial v}{\partial y} = 1 
\frac{\partial v}{\partial z} = 0 
\)

45. \( \frac{\partial w}{\partial x} = 0 
\frac{\partial w}{\partial y} = 0 
\frac{\partial w}{\partial z} = 1 
\)

47. \( \frac{\partial x}{\partial y} = 0 
\frac{\partial x}{\partial z} = 0 
\frac{\partial x}{\partial w} = 1 
\)

49. \( \frac{\partial y}{\partial x} = 0 
\frac{\partial y}{\partial y} = 1 
\frac{\partial y}{\partial z} = 0 
\)

51. \( \frac{\partial z}{\partial x} = 0 
\frac{\partial z}{\partial y} = 0 
\frac{\partial z}{\partial w} = 1 
\)
29. (a) The gradient points in the direction of greatest increase of the function.
(b) No.

\[ \begin{align*}
1. & \quad \frac{\partial^2 f}{\partial x^2} = 2(3 \sin 2x + 2 \cos 2x) + 2(x^3 \cos x + 3 \sin x) \\
3. & \quad \frac{\partial^2 f}{\partial y^2} = 7x^2 - 3y^2 \\
5. & \quad \frac{\partial^2 f}{\partial x \partial y} = 3(y - 3) + \sqrt{3} \left( z - \sqrt{3} \right) = 0 \\
7. & \quad \frac{\partial^2 f}{\partial z^2} = 2 \left( x - 2 \right) + 3 \left( y - 3 \right) + \sqrt{3} \left( z - \sqrt{3} \right) = 0 \\
9. & \quad \frac{\partial^2 f}{\partial x \partial y} = 3(y - 3) + \sqrt{3} \left( z - \sqrt{3} \right) = 0
\end{align*} \]
21. \( f \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) = 10 + \sqrt{2} \) is the maximum value;  
\( f \left( -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right) = 10 - \sqrt{2} \) is the minimum value.

23. \( f \left( \frac{3\sqrt{2}}{2}, -\frac{3}{2} \right) \approx 20.6913 \) is the maximum value;  
\( f(-2, -1) = -3 \) is the minimum value.

25. \( f \left( \frac{-2}{\sqrt{3}}, \frac{8}{\sqrt{3}} \right) = 29.9443 \) is the maximum value;  
\( f(x, 1 - x) = 0 \) for \( -\frac{1}{2} \leq x \leq \frac{1}{2} \).


1. (a) \( \{(x, y); x^2 + 4y^2 \leq 100\} \)  (b) \( \{(x, y); 2x - y \leq 1\} \)

3. \( 12x^2y^2 + 14xy^2 + 4y^2 + 24x^2y + 98xy^6 \)

5. \( e^x \sec^2 x; 2e^x \tan x \tan x; e^x \sec^2 x \)

7. \( 436a^3b^3 - 42y^6 \)

13. (a) \( -4i - j + k \); (b) \( -4 \cos \theta + \sin \theta \cos \theta - \cos \theta \)

15. \( \sqrt{3} + 2 \)

17. (a) \( x^2 + 2y^2 = 18 \); (b) \( 4i + 2j \);
15. \( k; 2k \)

17. The density is proportional to the squared distance from the origin; 
\[
\frac{25596a^4}{35} \left( 0, \frac{450}{79} \right)
\]

19. The density is proportional to the distance from the origin; 
\[
\frac{26b^4}{3} \left( 0, \frac{60}{13\pi} \right)
\]

21. \( r = \sqrt{5/12a} = 0.6455a \)

23. \( I_x = \frac{\pi b^4}{4}; I = a/2 \)

25. \( 3\pi b^4/4 \)

27. \( x = 0, y = (-15\pi + 32)a/(6\pi + 48) \)

29. (a) \( a^2 \); (b) \( 7\pi/2 \); (c) \( 11a^2/16 \)

31. \( I_x = \frac{\pi b^4}{2}; I_y = \frac{17\pi b^4}{2}; I_z = \frac{9\pi b^4}{2} \)

1. \( \sqrt{6}/3 \)

3. \( \pi /3 \)

5. \( 9 \sin^{-1}(\frac{1}{3}) \)

7. \( 8\sqrt{2} \)

9. \( 4\pi a(\sqrt{a^2 - b^2}) \)

11. \( 2\pi^2(a - 2) \)

13. \( \frac{\pi a^2(5\sqrt{5} - 1)}{6} \)

15. \( \frac{(17\pi - 1)}{6} \)

17. \( \frac{D^2\sqrt{a^2 + b^2} + c^2}{2\pi AC} \)

19. \( (h_1 + h_2)/2 \)

21. \( A = \pi b^2; B = 2\pi a(1 - \cos(b/a)); C = \pi b^2; D = \pi b^2(2a(1 + \sqrt{a^2 - b^2})) \)

23. \( a = 29,3297 \); (b) 15.4233

25. \( E/F, G/H, I \)

27. \( E/F, G/H, I \)

29. \( a = 29,3297; b = 15.4233 \)

31. \( E/F, G/H, I \)

23. \( 2\pi \int_{0}^{1} \sqrt{x^2 - y^2} \int_{0}^{x} dz \, dx = 2 \)

25. \( x = y = z \)

27. \( x = y = z = 3a/8 \)
29. $\int_{0}^{1} \int_{0}^{1} f(x, y, z) \, dy \, dx$  
31. $\int_{0}^{1} \int_{0}^{1} f(x, y, z) \, dx \, dz$  
33. $4$
35. Ave $T = 29.54$  
37. $(X, Y, Z) = \left( \frac{5}{16}, \frac{5}{16}, \frac{5}{16} \right)$
39. $(X, Y, Z) = \left( \frac{5}{16}, \frac{5}{16}, \frac{5}{16} \right)$
43. (a) $x = \frac{1}{288}$  
(b) $x = \frac{26}{27}$  
(c) $9$
45. (a) $7$  
(b) $\frac{1}{4}$  
(c) $5$
47. $x^2/576, 0 \leq x \leq 12.9$

1. True  
3. True  
5. True  
7. False  
9. True  
11. True  
13. False  
15. True  
17. False
19. $0$
21. $2 \pi$
25. (a) $g(u, v) = \begin{cases} e^u, & \text{if } 0 \leq u \leq 1 \\ 0, & \text{otherwise} \end{cases}$  
(b) $g_2(u) = \begin{cases} e^u, & \text{if } 0 \leq u \\ 0, & \text{otherwise} \end{cases}$
11. \( V_f(x, y) = (x \cos x + y \sin x)i + (x \cos y - y \sin y)j \)
13. \( V_f(x, y) = 2x \hat{i} + 2y \hat{j} + 2z \hat{k} \)
15. \( V/(x, y, z) = (y + z)i + (x + z)j + (x + y)k \)
17. \( \frac{\pi}{2} \) 19. \( \frac{3\pi}{4} \) 21. \( \frac{14\pi}{3} \)
23. The volume in problem 22 is that of a spherical shell centered at \((0, 0, 0)\) with outer radius = 2 and inner radius = 1.

\[ \left( \frac{3}{2} \right) \]

25. Work along \( C_1 \) is positive; work along \( C_2 \) is negative; work along \( C_3 \) is zero.

27. 2.25 gal
29. 2\( \pi \)m²
31. 4\( \pi \)²
33. (a) 27; (b) \(-29\pi/2\)
35. 6

7. \( (2i - 3y)\hat{i} - 3y\hat{j} + 2\hat{k} \)
9. \( x^2 + y^2 + z^2 \)
11. \( x^2 \cos z + y^2 \cos z - 2z^2 \sin z \)
15. 0
17. 2\( \pi \) cos y + 1; \( 2\pi \) sin z
19. (a) Meaningless; (b) vector field; (c) vector field; (d) scalar field; (e) vector field; (f) vector field; (g) vector field; (h) meaningless; (i) meaningless; (j) scalar field; (k) meaningful.
25. (a) div \( F \) = 0, div \( G \) < 0, div \( H \) = 0, div \( L \) > 0; (b) clockwise for \( H \), not at all for others.
(c) div \( F \) = 0, curl \( F \) = 0, div \( G \) = \(-2\pi \), curl \( G \) = 0, div \( H \) = 0, curl \( H \) = \(-2\pi \), div \( L \) = \( 1/\sqrt{3} \), curl \( L \) = 0
27. \( \text{div } \mathbf{F} > 0. A \text{ paddle wheel at the origin will not rotate.} \)
1. \( y = C_1 e^{2x} + C_2 e^{-3x} \)  
3. \( y = \frac{3}{8} x^2 - \frac{1}{2} e^{-3x} \)  
5. \( y = (C_1 + C_2 x) e^{3x} \)  
7. \( y = e^{3x} (C_1 e^{\sqrt{3}x} + C_2 e^{-\sqrt{3}x}) \)  
9. \( y = 3 \sin 2x + 2 \cos 2x \)  
11. \( y = e^{x} (C_1 \cos x + C_2 \sin x) \)  
13. \( y = C_1 + C_2 e^{-4x} + C_3 e^{x} \)  
15. \( y = C_1 e^{x} + C_2 e^{-x} + C_3 \cos 2x + C_4 \sin 2x \)  
17. \( y = D_1 \cosh 2x + D_2 \sinh 2x \)  
19. \( y = e^{x/2} [C_1 + C_2 x] \cos (\sqrt{3}/2)x \)  
21. \( y = \frac{1}{2} \left( C_1 + C_2 x \right) \sin (\sqrt{3}/2)x \) 
23. \( y = 0.5 \sin x + 0.5 \cos x \)  
25. \( y = 0.05 \times 10^{-2} \cos 377t \)  
27. \( y = 0.5 \cos 5t \)  
29. \( y = 0.5 m/s \)  
5. \( y = e^{0.05(x + 0.02 \sin 3x)} \)  
7. \( y = 14.4 s \)  
9. \( Q = 10^{-7}(1 - e^{-t}) \)  
11. (a) \( Q = 2.4 \times 10^{-6} \sin 377t \)  
(b) \( I = 9.05 \times 10^{-2} \cos 377t \)  
13. \( I = 12 \times 10^{-3} \sin 377t \)  
17. \( \theta = r \sin \theta \)  
19. \( y = 4 \)  
21. \( N = 4 \)  
23. \( P_{3i} P_{2j} P_{1k} \ldots \) are true  
25. \( P_{3i} P_{2j} P_{1k} \ldots \) are true  
27. True for all \( n \geq 1 \)  
29. \( P_{i} \) is true for all \( i \geq 1 \)  
31. True for \( n = 1, 2, 3, \ldots \) Proof is by induction.
functions defined by a table, 268
Parabolic Rule (Simpson's Rule), 265
Riemann sums, 260
Trapezoidal Rule, 264
Objective function, 151
Oblique asymptote, 25, 180
Odd-even trigonometric identities, 47
Odd function, 32
Old Goat Problem, 705
One-sided limits, 58-59, 66-68
One-to-one function, 332
Open interval, 8
Open set, 633
Operator, 107
Optical property:
of ellipse, 519-520
of hyperbola, 518-519
of parabola, 511
Ordered pair, 16, 139
Ordinary Comparison Test, 469-471
Orientation, curves, 295
Origin, 3, 16, 555
Orthogonal vectors, 686
Osculating plane, 600
Pappus, 312
Pappus's Theorem, 312-313, 322
Parabola, 26, 208-211
applications, 511
defined, 510
optical properties, 511
applications, 530
polar equation for, 541
standard equation, 510
Parabolic Rule (Simpson's Rule), 265-268, 263, 413
Paradox of Gabriel's horn, 249-250
Parallelogram, 556
Parametric equations, 294, 589
Parameter, 530
eliminating, 530-531
Parametric equations, 794-799
Parametric representation:
defined, 49
of curves in the plane, 530-534
cyldindrical, 531-532
differential, 530-531
Parametrization of a curve, 530
Parametric surface:
defined, 760
surface area for, 760-763
Perimeter probability density function, 441
Partial derivative symbol, 65
Partial derivatives, 654-667
genometric and physical interpretations, 625-626
mixture, 627
partial derivative of f with respect to x, 624, 627
second, 626
third, 636
Partial differential equations, 629
Partial fraction decomposition, 494-495
Partial sum, 455
Parratt, 224-230
regular, 227-229
Pascal, Blaise, 332, 304-305
Perrichon, 542
Period, 13
Periodic of trigonometric functions, 43-45
Periodicity, 257
Perpendicular lines, 21-22
Perpendicularity criterion, 567
Pickle, 19
Plane curve, 295, 530
arc length, 294-299
differential, 298-299
orientation, 295
Planes, 570-572
dot product, 570-572
linear equation, 571
modulating, 600
standard form (for the equation of a plane), 570
Point-slope form, 10-20
Polar coordinate system, 537-541
area in, 547-548
calculating, 547-550
intersection of curves in, 544
polar axis, 537
polar coordinates, defined, 537
polar equations, 548
for lines, circles, and conics, 540-541
pole (origin), 537
relation to Cartesian coordinates, 539-540
tangents to, 549-550
Polar coordinates:
and cylindrical coordinates, 714
double integrals in, 691-694
iterated integrals, 491-492
Polar equations:
graphs of, 542-544
cardioids, 543
lemniscates, 544
trochoids, 542-544
spirals, 544
intersection of curves in polar coordinates, 544
Polar resolute, 681
Polynomial functions, 28-39, 176-178, 630
continuity of, 63
Polynomials:
Machinism, 497-499
taylor, 497-499, 635-656
velocity, 83
as an accumulated velocity, 239
definite integral, 230
Position vector, 682
Positive series, 468-473
comparing a series with itself, 471-473
comparing one series with another, 469-471
Limit Comparison Test, 470
Ordinary Comparison Test, 469-470
Ratio Test, 471-473
Positively oriented, use of term, 735
Potential function, 732
Power function, 344
Power Rule, for derivatives, 108, 110, 133
Power series, 455, 479-482
convergence set, 480-481
operations on, 484-487
algebraic operations, 486-487
Vector-valued functions, 579-587
curvilinear motion, 581-585
differentiation formulas, 581
Kepler's Laws of planetary motion, 585-587
limit of, 579
Velocity, 126-128, 227, 582
definite integrals, 230
instantaneous, 93, 95-96
Vertex, 510
Vertical asymptote, 88
Vertical line, equation of, 20
Volume, 676
calculating, 682-683
of a coin, 281
solids, 282
Washers, method of, 284-285
Wave equation, 628
Weibull distribution, 441
Weierstrass, Karl, 67
Whispering gallery, 520
Work, 301–304, 736-740
application to pumping a liquid, 303-304
application to springs, 302-303
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<thead>
<tr>
<th>Mathematician</th>
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TEACHING OUTLINES
CALCULUS, Ninth Edition

The material in this section summarizes the content of each section (except the Review and Preview Problems and the Chapter Review), gives the theorems, and points out what should be remembered for future use. Specifically, we give the following for each section:

1. **Topics to Cover** The main topics are listed, along with suggestions on what to emphasize.

2. **Theorems** The titles (if any) of the theorems are given, along with a statement regarding our proof. Because many of the seemingly obvious theorems have difficult proofs, we do not prove all of the theorems in the book. In most cases, when we do not give a proof we give an intuitive explanation of the theorem’s result and we say that we are not going to prove the theorem. In many cases, we prove only one or two parts of a multipart theorem. There are several instances where students are asked to prove the theorem in the exercises.

3. **Homework** We suggest a homework assignment of moderate length. Often we give problems in pairs, an odd and an even problem. This way, students can attempt the odd problem and check their answer in the back of the book before they attempt an even problem for which they will not have the answer.

4. **Things to Remember** It isn’t always apparent to a student, or to a beginning teacher or teaching assistant, what material will be needed later in the book. We try to point out what topics will be needed and why they will be needed.

Experienced teachers may ignore much of the notes in this section. We hope that some of this material, such as the listing of theorems and their disposition, will be useful even to the most experienced teachers. To beginning teachers and teaching assistants the topics to cover and suggested homework may be helpful.

Students learn by doing. In calculus, this means working homework problems and writing their solutions. We encourage you to assign and collect homework assignments. Quick return of carefully graded homework can help students assess their progress in your course. You should demand quality writing from your students. Expect them to define their variables, honor the equal sign (i.e., say things are equal when they are, and don’t say things are equal when they aren’t), align the equal signs, explain their reasoning (in English!), and use parentheses when appropriate. Your work on the chalkboard can be a model for what you can expect of students. Emphasize continually the connection between geometry and algebra. Try to see every result from both perspectives. For example, the graph of \( y = x^2 \) increases without bound as \( x \) becomes large (geometric interpretation) and the quantity \( x^2 \) can be made large by taking the value of \( x \) to be large (algebraic interpretation).

The Review and Preview Problems provide the opportunity to have students review concepts needed in the upcoming chapters or to preview upcoming topics.

### Preliminaries

**0.1 Real Numbers, Estimation, and Logic**

*Topics to Cover* Number systems, including integers, rational numbers, and real numbers. Logic: "\( P \) implies \( Q \)," the converse and contrapositive. Order on the real line. Quantifiers "for all ..." and "there exists ..." Students should know most of the material in this section, although for many, the material on logic, or at least the terms converse and contrapositive, will be new.

*Theorems* None

*Homework* 1-6, 15, 17-22, 31, 37, 63-68

*Things to Remember* Properties of real numbers will be used throughout the course. Understanding the logic ("if then," contrapositive, converse) is key to understanding the theorems in the book.

**0.2 Inequalities and Absolute Values**

*Topics to Cover* Interval notation for closed, open, and half-open intervals. Solving inequalities. Absolute value function (emphasize that \(| b - a |\) represents the distance between \(a\) and \(b\) on the number line). Roots and the Quadratic Formula. Example 10 prepares students for the definition of the limit.

*Theorems* None

*Homework* 3-16, 23-32, 35-40, 49, 53

*Things to Remember* Interval notation will be used throughout the course. The ability to work with inequalities like \(2.9 < 1/x < 3.1\) will come in handy in Chapters 1 and 2.

**0.3 The Rectangular Coordinate System**

*Topics to Cover* Cartesian coordinates, the four quadrants, the Distance Formula, the equation of a circle, midpoint formula. Students will have seen many of the topics in this section. This section may be covered quickly or you may assign some problems and have the students read the section on their own.

*Theorems* None

*Homework* 3-10, 11-14, 17-20, 29-32, 55

*Things to Remember* Cartesian coordinates are used throughout the book.
0.4 Graphs of Equations

**Topics to Cover** Line, slope of a line, point-slope and slope-intercept form for the equation of a line, parallel and perpendicular lines. Graphing points, symmetry. Using technology (e.g., a graphing calculator or a computer algebra system) to plot equations. Shapes of quadratic and cubic graphs (Figure 6). Intersections of graphs. Students may have seen much of the material in this section. It may be helpful to point out that a graphing calculator or a CAS will graph a function in much the same way that we suggest in the text: Make a table and connect the points. Point out the inherent limitations in this method (e.g., with the function $f(x) = 1/x$ on the interval $[-2, 2]$).

**Theorems** None

**Homework** 1-8, 13-16, 31-34

**Things to Remember** Lines are one of the simplest geometrical objects that one can study. In Section 2.9 students will encounter the linear approximation to a function. In that section we emphasize that many functions will look linear when you zoom in on them. Graphing equations will be helpful throughout the course. Even with a graphing calculator, it is worthwhile to make some plots by hand; doing this often gives you some insight into the nature of the graph.

0.5 Functions and Their Graphs

**Topics to Cover** Definition of a function. Domain and range. Graphs of functions. Symmetry, even and odd functions.

**Theorems** None

**Homework** 1, 6, 7, 9, 10, 13-15, 20, 40, 41

**Things to Remember** Functions play a key role in calculus. For the first 10 chapters, the reader sees almost exclusively real-valued functions of one variable. Point out that the idea of a function can be generalized to include cases where the input is an ordered pair (or ordered $n$-tuple) and to cases where the output is an ordered pair (or ordered $n$-tuple).

0.6 Operations on Functions

**Topics to Cover** Adding, subtracting, multiplying, dividing functions. Composition of functions. Translations. Types of functions: constant, identity, polynomial, rational.

**Theorems** None

**Homework** 1, 5, 6, 11, 12-14, 17, 18, 23

**Things to Remember** In Section 2.5 when we cover the Chain Rule, students will need to understand function composition. Problems like 13 and 14 will help students in Section 2.5. Notation like $f^2(x)$ will be used throughout the book.

0.7 The Trigonometric Functions

**Topics to Cover** Definition of sine and cosine function. Properties and graphs of sine and cosine. Period and amplitude of sine and cosine functions. Modeling with the sine and cosine function. Other trigonometric functions: tangent, cotangent, secant, and cosecant functions. Most students will have seen the trigonometric functions, at least in their angle form. For some students, radians will be new. Emphasize that in calculus we will almost always deal with radian measure for angles.

**Theorems** None. The table on pp. 47-48 lists a number of trigonometric identities. These identities can also be found on the tear-out sheet from the back of the book.

**Homework** 1, 9, 11, 14, 16, 17, 25, 27, 32, 40, 42

**Things to Remember** Students should understand the six trigonometric functions with radian measure. Problems 40 (tangent of the angle between two lines) and 42 (area of a circular sector) are referred to later in the text.

1.1 Introduction to Limits

**Topics to Cover** An intuitive notion of limit. Graphical and tabular approaches to determining the limit. Left- and right-hand limits.

**Theorems** None

**Homework** 1-4, 7-26, 29-30, 45

**Things to Remember** Calculus is the study of limits, so an understanding of the concept of limit will be beneficial throughout the calculus sequence.

1.2 Rigorous Study of Limits

**Topics to Cover** Definition of the limit (Figure 3). Limit proofs (Optional).

**Theorems** None

**Homework** 1-4, 7-8, 11-12

**Things to Remember** Calculus is the study of limits, so an understanding of the concept of limit will be beneficial throughout the calculus sequence.

1.3 Limit Theorems

**Topics to Cover** Theorem A gives 9 statements regarding the limit of a sum, difference, product, and so on. This theorem is referred to a number of times throughout the book. The proof of Theorem A is optional. Evaluating limits using the Main Limit Theorem. The Squeeze Theorem.
Theorems

Theorem A Main Limit Theorem
Parts 1-5 are proved in the text. Problems 35 and 36 ask the student to prove parts 6 and 7. The others are proved in the Appendix. This theorem is cited countless times throughout the text.

Theorem B Substitution Theorem
The proof of Theorem B follows from repeated applications of Theorem A.

Theorem C (Functions that agree around c have the same limit at c.)
An intuitive explanation comes before the statement of the theorem.

Theorem D Squeeze Theorem
An $\epsilon-\delta$ proof is given, but it is optional.
All theorems in this section are also valid for right- and left-hand limits.

Homework 1-20, 25-26, 41-44

Things to Remember Use the Main Limit Theorem to evaluate limits.

1.4 Limits Involving Trigonometric Functions

Topics to Cover Limits of trigonometric functions. Combining the results of this section with the Main Limit Theorem from Section 1.3.

Theorems

Theorem A Limits of Trigonometric Functions
Proofs are given for limits of the sine and cosine functions. The proofs of the others are left as exercises (Problems 21 and 22).

Theorem B Special Trigonometric Limits
\[
\lim_{t \to 0} \frac{\sin t}{t} = 1 \quad \text{and} \quad \lim_{t \to 0} \frac{1 - \cos t}{t} = 0
\]

Both statements are proved using a geometric argument.

Homework 1-16

Things to Remember Theorem A says, in essence, that the trigonometric functions are continuous on their domains, a result that will be covered in Section 1.6. The special trigonometric limits will be needed in the next chapter when we discuss the derivatives of sine and cosine.

1.5 Limits at Infinity; Infinite Limits

Topics to Cover Definition of limits as \( x \to \infty \) or \( x \to -\infty \) and examples. Definitions of \( \lim_{x \to \infty} f(x) = \infty \) and \( \lim_{x \to -\infty} f(x) = -\infty \). Asymptotes. Sequences and limits of sequences (which are needed in Section 3.7 and Chapter 4).

Theorems None

Homework 1-10, 21-24, 27-30, 43-46

Things to Remember Chapter 8, on l'Hôpital's Rule and improper integrals, uses the ideas of limits at infinity. Asymptotes are helpful in graphing a function.

1.6 Continuity of Functions

Topics to Cover Definition of continuity at a point, continuity of polynomial and rational functions, continuity under function operations. Definition of continuity on an open interval and on a closed interval. Intermediate Value Theorem.

Theorems

Theorem A Continuity of Polynomial and Rational Functions
The proofs follow directly from Theorem 1.3B.

Theorem B Continuity of Absolute Value and nth Root Functions
The proofs follow directly from Theorem 1.3A.

Theorem C Continuity under Function Operations
A proof is given for the product. The proofs of the others follow from Theorem 1.3A.

Theorem D Continuity of Trigonometric Functions
The proof is given for the sine, cosine, and tangent. A similar argument applies to the other trigonometric functions.

Theorem E Composite Limit Theorem
The proof, which is optional, is given.

Theorem F Intermediate Value Theorem
Homework 1-10, 18-21, 24-30, 51-53, 63

Things to Remember The concept of continuity. The Intermediate Value Theorem is used occasionally in the rest of the text.

The Derivative

2.1 Two Problems with One Theme

Topics to Cover Slope of the tangent line. Average and instantaneous velocity. Rate of change.

Theorems None

Homework 1-4, 9, 13, 17, 25, 27

Things to Remember Instantaneous rate of change is the limit of a quotient.

2.2 The Derivative

Topics to Cover Definition of the derivative, examples of finding derivatives from the definition. Equivalent forms of the derivative. A differentiable function must be continuous, but not conversely.

Theorems

Theorem A Differentiability Implies Continuity
The proof is given.
Homework 1-8, 17-20, 27, 37-40

Things to Remember  The derivative is an instantaneous rate of change, that is, a limit of a difference quotient.

2.3 Rules for Finding Derivatives

Topics to Cover  Rules for finding derivatives: power, sum, difference, product, and quotient rules. Examples.

Theorems

Theorem A  Constant Function Rule
Theorem B  Identity Function Rule
Theorem C  Power Rule (Positive Integral Exponents)
Theorem D  Constant Multiple Rule
Theorem E  Sum Rule (Derivative of a Sum Is the Sum of Derivatives)
Theorem F  Difference Rule (Derivative of a Difference Is the Difference of Derivatives)
Theorem G  Product Rule
Theorem H  Quotient Rule

Theorems A, B, C, D, E, G, and H are proved. The proof of theorem F is left as an exercise (Problem 54).

Homework 1-20, 31-36, 49-52

Things to Remember  All of the rules of this section are needed in the rest of the book. Despite all of these rules, don’t let students forget that the derivative is still the limit of a quotient and therefore is an instantaneous rate of change.

2.4 Derivatives of Trigonometric Functions

Topics to Cover  Derivative formulas for sine and cosine (derivations require Theorem 1.4B. Special Trigonometric Limits). Derivative formulas for tangent, cotangent, secant, and cosecant (derivations require rules for derivatives from Section 2.3).

Theorems

Theorem A (Derivatives of Sine and Cosine)
The derivations of these two formulas are given before the theorem is stated.

Theorem B (Derivatives of Tangent, Cotangent, Secant, and Cosecant)
We say that the formulas for these derivatives can be obtained by applying the quotient rule, but the details are left as exercises (Problems 5-8).

Homework 1-18, 21, 23, 25

Things to Remember  Although the rules for derivatives of sine and cosine are easy to remember, and are certainly the most important, the derivatives of the other four trigonometric functions should also be committed to memory.

2.5 The Chain Rule

Topics to Cover  Operators and the \( D_x \) notation. Chain Rule using prime notation and operator notation. Examples. Applying the Chain Rule more than once.

Theorems

Theorem A  Chain Rule
A partial proof of the chain rule is given in Section 2.5. A complete proof is given in the Appendix.

Homework 1-28, 33-36, 41-42, 63-65

Things to Remember  The Chain Rule will be needed for almost every derivative from here on out. Students should know the Chain Rule so well that they can use it in reverse when they study substitution for indefinite integrals (Chapter 4).

2.6 Higher-Order Derivatives

Topics to Cover  Definitions of second-, third-, and higher-order derivatives. Prime, operator, Leibniz notation for higher-order derivatives. Velocity and acceleration as first and second derivatives, respectively, of position. Mathematical modeling with derivatives.

Theorems

None

Homework 1-14, 20, 21, 23-25, 31

Things to Remember  Definitions of higher-order derivatives and their representations in prime, operator, and Leibniz notation. Acceleration is the rate of change of velocity, which is the rate of change of position. Therefore, acceleration is the second derivative of position.

2.7 Implicit Differentiation

Topics to Cover  Functions defined implicitly. Differentiation of functions defined implicitly. Examples. The Power Rule for rational exponents.

Theorems

Theorem A  Power Rule (Rational Exponents)
A partial proof, using implicit differentiation, is given. This proof assumes the differentiability of \( y = x^{p/q} \), a result not yet established. We give a complete proof in the Appendix.

Homework 1-8, 13-16, 21-26, 37-38

Things to Remember  Implicit differentiation is needed in the next section on related rates, in Chapter 6 where we derive formulas for the derivatives of the inverse trigonometric functions, and in a few other places.

2.8 Related Rates

Topics to Cover  Rates of change. Related rates. Examples. The systematic procedure.

Theorems

None

Homework 1-4, 7, 8, 11, 12, 15, 16, 19, 20

Things to Remember  The method of related rates is not used much in the remainder of the book, but it is a good application of differentiation and it reinforces the idea that
the derivative is a rate of change. These problems are also good for developing problem-solving skills.

2.9 Differentials and Approximations
Topics to Cover Differentials. Relation to differentiation. Approximations. Absolute and relative errors.

Theorems None

Homework 1–6, 10, 14, 16–19, 23–24, 37–40

Things to Remember Students should remember to distinguish between derivatives and differentials. They will often write dy when they mean dy/dx.

Applications of the Derivative

3.1 Maxima and Minima
Topics to Cover Definitions of maximum, minimum, extremum, objective function, critical point, stationary point, singular point. Examples.

Theorems

Theorem A Max-Min Existence Theorem
The proof is difficult, so it is omitted.

Theorem B Critical Point Theorem
The proof is given.

Homework 1–2, 5–14, 29–31

Things to Remember How to find the maximum or minimum of a function.

3.2 Monotonicity and Concavity
Topics to Cover Definitions of increasing, decreasing, monotonic, concavity. Examples. Theorems on monotonicity and concavity.

Theorems

Theorem A Monotonicity Theorem
The proof follows directly from Theorem A.

Homework 1–8, 11–14, 19–24, 29–32

Things to Remember The maximum of a function is a number in its range; an inflection point is an ordered pair.

3.3 Local Maxima and Minima
Topics to Cover Definitions of local maximum, local minimum, local extremum. Where do local extrema occur? First derivative test, second derivative test. Examples.

Theorems

Theorem A First Derivative Test
The proof of part (i) is given. Parts (ii) and (iii) are similar.

Theorem B Second Derivative Test
The proof is given.

Homework 1–6, 11–14, 21–24, 31–32, 35–38

Things to Remember First and second derivative tests for local extrema.

3.4 Practical Problems

Theorems None

Homework 1, 2, 10–12, 17–20, 25–28, 41, 43

Things to Remember The important skill for this section is translating a problem to mathematical terms so we can use the methods of calculus.

3.5 Graphing Functions Using Calculus
Topics to Cover Using information regarding increasing/decreasing, concave up/down, intercepts, max-min, inflection points, and so on to sketch the graph of a function.

Horizontal, vertical, and oblique asymptotes.

Theorems None

Homework 1–6, 13–14, 17, 23, 31–32, 34

Things to Remember How the properties of a function (e.g., increasing, concave down, etc.) affect its graph.

3.6 The Mean Value Theorem for Derivatives
Topics to Cover The Mean Value Theorem for Derivatives. Proof and illustration of the theorem.

Theorems

Theorem A Mean Value Theorem for Derivatives
The proof is given. The proof of the Monotonicity Theorem (Theorem 3.2A) is also given in this section.

Theorem B \( F'(x) = G'(x) \Rightarrow F(x) = G(x) + C \)

The proof is given.

Homework 1–6, 18–21, 31–32

Things to Remember The geometric interpretation of the Mean Value Theorem for Derivatives. Functions with the same derivative differ by a constant.

3.7 Solving Equations Numerically
Topics to Cover Bisection Method, Newton’s Method.
The fixed-point algorithm. Examples.

Theorems None

Homework 1–3, 5, 8, 12, 13, 17, 21, 25–26, 31–33

Things to Remember The advantages and disadvantages of the three methods. Some formulations of \( x = g(x) \) will lead to convergence to a fixed point, and others will not.

3.8 Antiderivatives
Topics to Cover Definition of antiderivative. Power Rule and Generalized Power Rule.
the derivative is a rate of change. These problems are also good for developing problem-solving skills.

2.9 Differentials and Approximations
Topics to Cover  Differentials. Relation to differentiation. Approximations. Absolute and relative errors.

Theorems  None

Homework  1–6, 10, 14, 16–19, 23–24, 37–40

Things to Remember  Students should remember to distinguish between derivatives and differentials. They will often write \( dy \) when they mean \( \frac{dy}{dx} \).

Applications of The Derivative

3.1 Maxima and Minima
Topics to Cover  Definitions of maximum, minimum, extremum, objective function, critical point, stationary point, singular point. Examples.

Theorems  
- **Theorem A** Max-Min Existence Theorem
  - The proof is difficult, so it is omitted.
- **Theorem B** Critical Point Theorem
  - The proof is given.

Homework  1–2, 5–14, 29–31

Things to Remember  How to find the maximum or minimum of a function.

3.2 Monotonicity and Concavity
Topics to Cover  Definitions of increasing, decreasing, monotonic, concavity. Examples. Theorems on monotonicity and concavity.

Theorems  
- **Theorem A** Monotonicity Theorem
  - The proof follows directly from Theorem A.
- **Theorem B** Concavity Theorem
  - The proof is given.

Homework  1–8, 11–14, 19–24, 29–32

Things to Remember  Definitions. The maximum of a function is a number in its range; an inflection point is an ordered pair.

3.3 Local Maxima and Minima
Topics to Cover  Definitions of local maximum, local minimum, local extremum. Where do local extrema occur? First derivative test, second derivative test. Examples.

Theorems  
- **Theorem A** First Derivative Test
  - The proof of part (i) is given. Parts (ii) and (iii) are similar.
- **Theorem B** Second Derivative Test
  - The proof is given.

Homework  1–6, 11–14, 21–24, 31–32, 35–38

Things to Remember  First and second derivative tests for local extrema.

3.4 Practical Problems

Theorems  None

Homework  1, 2, 10–12, 17–20, 25–28, 41, 43

Things to Remember  The important skill for this section is translating a problem to mathematical terms so we can use the methods of calculus.

3.5 Graphing Functions Using Calculus
Topics to Cover  Using information regarding increasing/decreasing, concave up/down, intercepts, max-min, inflection points, and so on to sketch the graph of a function. Horizontal, vertical, and oblique asymptotes.

Theorems  None

Homework  1–6, 13–14, 17, 23, 31–32, 34

Things to Remember  How the properties of a function (e.g., increasing, concave down, etc.) affect its graph.

3.6 The Mean Value Theorem for Derivatives
Topics to Cover  The Mean Value Theorem for Derivatives. Proof and illustration of the theorem.

Theorems  
- **Theorem A** Mean Value Theorem for Derivatives
  - The proof is given. The proof of the Monotonicity Theorem (Theorem 2.4A) is also given in this section.
- **Theorem B** \( F'(x) = G'(x) \Rightarrow F(x) = G(x) + C \)

The proof is given.

Homework  1–6, 18–21, 31–32

Things to Remember  The geometric interpretation of the Mean Value Theorem for Derivatives. Functions with the same derivative differ by a constant.

3.7 Solving Equations Numerically
Topics to Cover  Bisection Method, Newton’s Method. The fixed-point algorithm. Examples.

Theorems  None

Homework  1–3, 5, 8, 12, 13, 17, 21, 25–26, 31–33

Things to Remember  The advantages and disadvantages of the three methods. Some formulations of \( x = g(x) \) will lead to convergence to a fixed point, and others will not.

3.8 Antiderivatives
Topics to Cover  Definition of antiderivative. Power Rule and Generalized Power Rule.
4.5 The Mean Value Theorem for Integrals and the Use of Symmetry
Topics to Cover

Theorems
Theorem A Mean Value Theorem for Integrals
Theorem B Symmetry Theorem
Theorem C (For a function $f$ with period $p$, $\int_{a}^{b} f(x) \, dx = \int_{0}^{p} f(x) \, dx$)

The proofs for Theorems A and B are given. The proof of Theorem C for even functions is given; the proof for odd functions is an exercise.

Homework
1-8, 15-20, 29, 35-41, 48

Things to Remember
Figures 3 and 6-8.

4.6 Numerical Integration
Topics to Cover
Examples where the Second Fundamental Theorem of Calculus can’t be applied. Riemann sums as approximations. Trapezoidal Rule and its error formula. Parabolic Rule and its error formula. Functions defined by a table.

Theorems
Theorem A Errors for the Five Numerical Methods
The proof of Theorem A is difficult and is omitted.

Homework
1-4, 7-8, 11, 15, 19-22, 27-28

Things to Remember
The Parabolic Rule can be used to approximate the value of a definite integral for which you might not be able to apply the Second Fundamental Theorem of Calculus. Many problems in subsequent chapters ask the student to use the Parabolic Rule with $n = 8$ or $n = 10$ to approximate integrals that cannot be evaluated with the Second Fundamental Theorem of Calculus.

Applications of the Integral

5.1 The Area of a Plane Region
Topics to Cover
Area under a curve and above the x-axis. Regions below the x-axis. Regions between curves. Slice, approximate, integrate. Distance and displacement.

Theorems None

Homework
1-8, 11-20, 27-29, 33, 34

Things to Remember
Slice, approximate, integrate.

5.2 Volumes of Solids: Slabs, Disks, Washers
Topics to Cover
Volumes of cylinders. Solids of revolution: disks, washers. Other solids with known cross-sectional areas.

Theorems None

Homework
1-8, 11-12, 17-18, 27, 33, 40-41

Things to Remember
The idea of slice, approximate, integrate can be used to find a number of quantities.

5.3 Volumes of Solids of Revolution: Shells
Topics to Cover

Theorems None

Homework
1-8, 13-16, 19-21

Things to Remember
Slice, approximate, integrate.

5.4 Length of a Plane Curve
Topics to Cover

Theorems None

Homework
1-4, 7-8, 13, 17, 18, 23

Things to Remember
$ds = \sqrt{1 + (y')^2} \, dx$ and the other similar results are needed later in the book.

5.5 Work and Fluid Force
Topics to Cover
Definition of work as $F \cdot D$ for a constant force, and the integral of force times distance for a variable force. Application to springs and to pumping a liquid. Fluid force.

Theorems None

Homework
1-2, 5, 9-10, 25-28

Things to Remember
Work, as $F \cdot D$, is needed in Chapters 11 and 14.

5.6 Moments, Center of Mass
Topics to Cover

Theorems
Theorem A Pappus’s Theorem
The proof is left as an exercise (Problem 28).

Homework
1-2, 7-15, 26-27

Things to Remember
Moments and center of mass. These ideas arise again in Chapter 13.

5.7 Probability and Random Variables
Topics to Cover
Discrete probability. Continuous distribution of probability. CDFs and PDFs. Computing probabilities and means (expectations) by integration.

Theorems
Theorem A Properties of PDFs and CDFs
The proofs are left as an exercise (Problem 19).
Homework 1-2, 9-12, 20
Things to Remember: Emphasize connection between probability and means of random variables to mass and center of mass.

Transcendental Functions

6.1 The Natural Logarithm Function
Topics to Cover
Is there a function whose derivative is \(1/x\)?
Definition of the natural logarithm as an accumulation function.
Derivative of natural logarithm function.
Examples. Properties of the natural logarithm function.
Logarithmic differentiation.

Theorems

- **Theorem A** (Properties of the Natural Logarithm: \(\ln 1 = 0, \ln ab = \ln a + \ln b, \text{ etc.}\))
  All four parts are proved.

Homework 1, 3-10, 15-22, 30-32, 35
Things to Remember: The natural logarithm function, defined as an accumulation function, is a function whose derivative is \(1/x\).

6.2 Inverse Functions and Their Derivatives
Topics to Cover
Definition and existence of the inverse of a function. Derivatives of inverse functions.

Theorems

- **Theorem A** (A Monotonic Function Has an Inverse.)
- **Theorem B** Inverse Function Theorem
  The proof of Theorem A is given. The proof of Theorem B is difficult and is omitted.

Homework 1, 2, 7-12, 15-18, 29, 33, 43
Things to Remember: Inverses exist for monotone functions.

6.3 The Natural Exponential Function
Topics to Cover
Definition of natural exponential function as inverse of natural logarithm function. The number \(e\).
Properties of the exponential function.

Theorems

- **Theorem A** \(e^{x+y} = e^x e^y\)
The first statement is proved; the proof of the second is similar.

Homework 3-8, 11-21, 25-29, 37-44
Things to Remember: The natural exponential function is the inverse of the natural logarithm function and is \(e^x\), where \(e\) is the real number satisfying \(\ln e = 1\).

6.4 General Exponential and Logarithmic Functions
Topics to Cover
Definition of \(a^x\). Properties of \(a^x\). Definition and properties of \(\log_a x\). Comparison of \(a^x, x^a, \) and \(x^x\).

Theorems

- **Theorem A** Properties of Exponents \((a^p a^q = a^{p+q}, \text{ etc.})\)
  Two parts are proved. The proofs of the rest are left as exercises.

- **Theorem B** Exponential Function Rules (Derivative and Integral of Exponential Functions)
  The derivative formula is proved; the integral formula follows immediately from the derivative formula.

Homework 1-5, 9-10, 13-16, 17-26, 21, 33, 35-36
Things to Remember: We have come full circle (Figure 1 in the text). Our definition of \(\ln x\) as an accumulation function has led us to \(\ln x = \ln a\).

6.5 Exponential Growth and Decay
Topics to Cover
Differential equations. Solving differential equations by separation of variables. Exponential growth and decay, compound interest.
Examples.

Theorems

- **Theorem A** \(\lim_{h \to 0} (1 + h)^{1/h} = e\)
The proof is given.

Homework 1-3, 8, 9, 12, 13-15, 19-21, 26, 38
Things to Remember: Simple differential equations like \(y' = ky\) lead to exponential growth or decay.

6.6 First-Order Linear Differential Equations
Topics to Cover
Classification of differential equations.
Solving first-order linear equations using integrating factors. Examples and applications.

Theorems: None

Homework 1-5, 15-17
Things to Remember: Multiplying both sides by the integrating factor is just what is needed to make the left-hand side the derivative of a product. This trick allows us to integrate both sides and thereby solve the equation.

6.7 Approximations for Differential Equations
Topics to Cover
Slope fields. Euler's method. The effect of the step size \(h\).

Theorems: None

Homework 1, 3, 7, 11, 14, 17
Things to Remember: Slope fields can give us a general idea about what solutions of a differential equation will look like. Euler's method can give an approximation to the solution of a first-order differential equation given an initial condition.

6.8 The Inverse Trigonometric Functions and Their Derivatives
Topics to Cover
To obtain an inverse for the trigonometric functions, we must restrict the domain. Inverses of the
six trigonometric functions. Derivatives of trigonometric and inverse trigonometric functions.

**Theorems**

- **Theorem A** \( \sin(\cos^{-1} x) = \sqrt{1 - x^2}, \) etc.
  The proof of part (i) is given; the others are similar.

- **Theorem B** Derivatives of Four Inverse Trigonometric Functions (\( \sin^{-1}, \cos^{-1}, \tan^{-1}, \) and \( \sec^{-1} \))

A proof is given for the inverse sine function. Proofs for the inverse cosine and inverse tangent are similar. The proof for the inverse secant involves a twist, and it is given.

**Homework** 1–5, 11–13, 25, 29, 39–50, 55–63, 73, 87

**Things to Remember** Restricting the domain of the trigonometric functions allows us to obtain inverse functions. Remember the formulas for the derivatives of the trigonometric and inverse trigonometric functions.

### 6.9 The Hyperbolic Functions and Their Inverses

**Topics to Cover** Definitions and derivatives of the six hyperbolic functions. Inverse hyperbolic functions. Applications.

**Theorems**

- **Theorem A** Derivatives of Hyperbolic Functions

  The derivatives of \( \sinh x \) and \( \cosh x \) are derived. The derivatives of the others are just straightforward applications of the Quotient Rule.

**Homework** 1–3, 13–28, 38–43, 55

**Things to Remember** Definitions and derivatives of hyperbolic functions.

### Techniques of Integration

#### 7.1 Basic Integration Rules

**Topics to Cover** Standard integral forms (students should commit these to memory). Substitutions in indefinite integrals. Examples. Substitutions in definite integrals.

**Theorems**

- **Theorem A** Substitutions in Indefinite Integrals

  This is just a restatement of Theorem 4.4B.

**Homework** 1–24, 41–47, 55, 56

**Things to Remember** Substitution, combined with the standard integral forms, is a powerful tool for evaluating integrals. A substitution in a definite integral requires three things:

1. substitution in the integrand
2. substitution for the differential
3. changing the limits of integration

#### 7.2 Integration by Parts

**Topics to Cover** Integration by parts for indefinite and definite integrals. Repeated integration by parts. Reduction formulas. Examples.

**Theorems** None.

**Homework** 1–16, 25, 29, 37–42, 55, 62

**Things to Remember** Next to substitution, integration by parts is the most frequently used technique of integration.

#### 7.3 Some Trigonometric Integrals

**Topics to Cover** Review some trigonometric identities. Integrals of the type

1. \( \int \sin^n x \cos^m x \, dx, \) \( \int \sin^n x \, dx, \) \( \int \cos^n x \, dx, \)
2. \( \int \tan^m x \sec^n x \, dx, \) \( \int \tan^m x \, dx, \) \( \int \cot^m x \csc^n x \, dx, \)
3. \( \int \tan^m x \sec^n x \, dx, \) \( \int \sec^m x \, dx, \)
4. \( \int \tan^m x \, dx, \) \( \int \cot^m x \, dx, \)
5. \( \int \tan^m x \csc^n x \, dx, \) \( \int \cot^m x \sec^n x \, dx, \)

Examples.

**Theorems** None

**Homework** 1–6, 19, 20, 29, 30

**Things to Remember** Remember the trigonometric identities and how they lead to evaluation of an integral.

#### 7.4 Rationalizing Substitutions

**Topics to Cover** Integrands involving \( \sqrt{ax + b} \). Integrands involving \( \sqrt{a^2 - x^2}, \) \( \sqrt{x^2 + a^2}, \) and \( \sqrt{x^2 - a^2}. \)

Completing the square. Examples.

**Theorems** None

**Homework** 1–10, 17–22, 32

**Things to Remember** Remember what substitutions work in each case.

#### 7.5 Integration of Rational Functions

**Topics to Cover** Long division of polynomials. Partial fraction decomposition. Examples.

**Theorems** None

**Homework** 1–10, 17–20, 31–32, 40, 49

**Things to Remember** Partial fraction decomposition is a useful technique for solving differential equations.

#### 7.6 Strategies for Integration

**Topics to Cover** Review techniques of integration. Using tables of integrals. Using a CAS or graphing calculator. Approximate versus exact values for definite integrals. Solving equations involving the upper limit of an integral.

**Theorems** None
T-10 Teaching Outlines

Homework 1-4, 13, 16, 21, 31-33, 42, 55

Things to Remember A table of integrals is almost always used in conjunction with the method of substitution. Emphasize the difference between the exact value of an integral and an approximation to it.

Indeterminate Forms and Improper Integrals

8.1 Indeterminate Forms of Type 0/0


Theorems

Theorem A L'Hôpital's Rule
The proof is given after Cauchy's Mean Value Theorem.

Theorem B Cauchy's Mean Value Theorem
The proof is given.

Homework 1-12, 19-21, 29

Things to Remember When and how to apply L'Hôpital's Rule.

8.2 Other Indeterminate Forms

Topics to Cover L'Hôpital's Rule for forms of type \( \infty/\infty \). The indeterminate forms \( 0^0 \), \( \infty^0 \), and \( 1^\infty \). Examples.

Theorems

Theorem A L'Hôpital's Rule for Forms of Type \( \infty/\infty \)
The proof is omitted, but a plausibility argument is given.

Homework 1-14, 19-22, 27, 35, 39, 44

Things to Remember When and how to apply L'Hôpital's Rule.

8.3 Improper Integrals: Infinite Limits of Integration


Theorems None

Homework 1-12, 19, 20, 28

Things to Remember An integrand with an infinite limit of integration is the limit of a proper integral.

8.4 Improper Integrals: Infinite Integrands

Topics to Cover Definition of integral over interval where the integrand is infinite at one end point. Integrands that are infinite at an interior point. Examples.

Theorems None

Homework 1-14, 21-25, 35-36

Things to Remember Apply the correct limit to evaluate an improper integral.

9.1 Infinite Sequences

Topics to Cover Notation and terminology for infinite sequences. Definitions of convergence and divergence.

Theorems

Theorem A Properties of Limits of Sequences
Theorem B Squeeze Theorem
The proofs of Theorems A and B are similar to the analogous theorems for limits of functions of a single variable.

Theorem C \( (\lim |a_n| = 0 \implies \lim a_n = 0) \)
The proof follows from the Squeeze Theorem.

Theorem D Monotonic Sequence Theorem
The proof is difficult, and so it is omitted.

Homework 1, 8, 19, 21-25, 31, 53

Things to Remember Definition of convergence of a sequence.

9.2 Infinite Series

Topics to Cover Definitions of infinite series, partial sums, convergence, and divergence. Examples. The harmonic series. Collapsing series. Linearity of \( \Sigma \) for convergent series.

Theorems

Theorem A nth Term Test for Divergence
The proof is given.

Theorem B Linearity of Convergent Series
This theorem is proved.

Theorem C \( (\Sigma a_k \text{ diverges and } c \neq 0 \implies \Sigma ca_k \text{ diverges}) \)
The proof is left as an exercise (Problem 41).

Theorem D Grouping Terms in an Infinite Series
A sketch of the theorem is given. It is assumed that if \( \{s_n\} \) is a subsequence of \( \{s_m\} \), and if \( s_m \to S \), then \( T_m \to S \) as well.

Homework 1-10, 15-16, 27, 29, 33

Things to Remember nth term test for divergence can be used to establish divergence only. (Students will often try to apply Theorem A to establish the convergence of a series.)

9.3 Positive Series: The Integral Test

Topics to Cover Bounded partial sums and the bounded sum test. Integral test, p-series.

Theorems

Theorem A Bounded Sum Test
The proof, which relies on Theorem 9.1D, is proved.

Theorem B Integral Test
The proof is given.
Homework 1-8, 13-20, 23-24, 27, 34

Things to Remember  The p-series converges for \( p > 1 \) and diverges for \( p \leq 1 \).

9.4 Positive Series: Other Tests
Topics to Cover  Comparison test. Limit comparison test.

Theorems
- **Theorem A**  Ordinary Comparison Test
  The proof is given.
- **Theorem B**  Limit Comparison Test
  Most parts of the theorem are proved. The proof of the last part is left as an exercise (Problem 37).
- **Theorem C**  Ratio Test
  This theorem is proved.

Homework  1-14, 21-23, 36
Things to Remember  At this point, students should review all tests for convergence or divergence.

9.5 Alternating Series, Absolute Convergence, and Conditional Convergence

Theorems
- **Theorem A**  Alternating Series Test
  The proof is given before the statement of the theorem.
- **Theorem B**  Absolute Convergence Test
  This theorem is proved.
- **Theorem C**  Absolute Ratio Test
  The proof is given.
- **Theorem D**  Rearrangement Theorem
  The proof is difficult and is omitted.

Homework  1-14, 29-30
Things to Remember  Definitions of absolute and conditional convergence, and how to test for these.

9.6 Power Series
Topics to Cover  Definitions of power series, convergence test. Possible types of convergence sets. Radius of convergence. Power series in \( x - a \).

Theorems
- **Theorem A**  (Convergence set of a power series is a point, an interval, or the whole real line.)
  The proof is given.
- **Theorem B**  (A power series converges absolutely on the interior of its interval of convergence.)
  The proof is contained within the proof of Theorem A.

Homework  1-14, 24, 28, 31
Things to Remember  A power series is a series of functions of \( x \). Rather than asking whether a series converges, we ask for what values of \( x \) it converges.

9.7 Operations on Power Series
Topics to Cover  Term-by-term differentiation and integration. Examples based on the geometric series. Multiplying and dividing power series.

Theorems
- **Theorem A**  (On the interior of the convergence set, power series can be differentiated and integrated term by term.)
- **Theorem B**  (Convergence of Sums, Differences, Products, and Quotients of Convergent Power Series)
  The proofs of Theorems A and B are difficult and are omitted.

Homework  1-8, 13-15, 17-18, 25
Things to Remember  Term-by-term differentiation and integration is allowed under the right conditions.

9.8 Taylor and Maclaurin Series

Theorems
- **Theorem A**  Uniqueness Theorem
  The proof is given before the statement of the theorem.
- **Theorem B**  Taylor's Formula with Remainder
  The proof is given for the case of \( n = 4 \). The proof for an arbitrary \( n \) is left as an exercise (Problem 37).
- **Theorem C**  Taylor's Theorem
  The proof follows directly from Theorem B.
- **Theorem D**  Binomial Series
  A partial proof is given. Problem 38 gives an entirely different way to prove the theorem.

Homework  1-4, 15-16, 29, 33
Things to Remember  Memorize the important Maclaurin series given at the end of the section.

9.9 The Taylor Approximation to a Function
Topics to Cover  Taylor polynomial of order 1 and of order \( n \). Maclaurin polynomials. Error in the method. Bounding the absolute value of the remainder. Determining the size of \( n \) to meet a specified error. The error of calculation.

Theorems  None
10.4 Parametric Representation of Curves in the Plane


Theorems

Theorem A \( \left( \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \right) \)

The proof is given.

Homework 1-4, 9-10, 17-18, 21-23, 33-35-39

Things to Remember: A general familiarity with curves defined parametrically is needed in the next chapter.

10.5 The Polar Coordinate System

Topics to Cover: Identifying points in the plane by \( r \) and \( \theta \). Polar equations. Graphs of polar equations. Relationship between Cartesian and polar coordinates. Polar equations for lines, circles, and other conics.

Theorems: None

Homework 5, 7, 11-12, 17-21, 23-30, 41

Things to Remember: Remember the relationships between polar and Cartesian coordinates. These will be needed again in Chapters 13 and 14.

10.6 Graphs of Polar Equations

Topics to Cover: Symmetry tests. Cardioids, limacons, lemniscates, roses, spirals. Intersections of curves in polar coordinates.

Theorems: None

Homework 1-14, 21-23, 27, 33-35

Things to Remember: This section should develop some general skills working with polar equations. This overall familiarity is more important than any specific skill.

10.7 Calculus in Polar Coordinates

Topics to Cover: Area of a sector. Area in polar coordinates. Tangents in polar coordinates.

Theorems: None

Homework 1-8, 14, 15, 18, 23, 24, 29

Things to Remember: Slice, approximate, and integrate worked again!

Geometry in Space and Vectors

11.1 Cartesian Coordinates in Three-Space

Topics to Cover: Right-handed coordinate systems. Distance Formula. Graphs in three-space, spheres, planes.

Theorems: None

Homework 1, 8, 13-15, 17-20, 25-28, 33, 41

Things to Remember: An overall familiarity with three-space is the goal of this section. Much of the rest of the book deals with calculus in three-space.

11.2 Vectors

Topics to Cover: Direction and magnitude of vectors. Sums, differences, and scalar multiples of vectors. Applications.

Theorems

Theorem A (Properties of Vectors)

The proofs are easy. We prove 6 and 9; other parts are left as exercises (Problems 25-26).

Homework 1, 5, 9, 11, 13, 16, 18, 20
Things to Remember  Students should understand the algebraic and geometric interpretation of vectors, as well as the properties of vectors.

11.3 The Dot Product

Theorems

Theorem A  Properties of Dot Product
These properties are easy to establish.

Theorem B  \((\mathbf{u} \cdot \mathbf{v} = ||\mathbf{u}|| ||\mathbf{v}|| \cos \theta)\)
The proof is given.

Theorem C  Perpendicularity Criterion
The proof is given.

Homework  1, 5, 8, 11, 13, 15–17, 23, 29, 43, 45, 65

Things to Remember  Students must see vectors from both a geometric and algebraic point of view. The formula for \(\cos \theta\) is needed in a number of places in the rest of the book.

11.4 The Cross Product

Theorems

Theorem A  (Properties of the Cross Product)
Properties 1 and 3 are proved. A plausibility argument is given for property 2, which is difficult to prove rigorously.

Theorem B  \((\mathbf{u} \times \mathbf{v} = 0)\)
The proof follows directly from Theorem A.

Theorem C  (Further Properties of the Cross Product)
The proofs are easy and are left as exercises.

Homework  1–3, 7, 9, 11–13, 17, 20, 25, 27

Things to Remember  The cross product \(\mathbf{u} \times \mathbf{v}\) is perpendicular to both \(\mathbf{u}\) and \(\mathbf{v}\). The area of a parallelogram whose sides are \(\mathbf{u}\) and \(\mathbf{v}\) is \(||\mathbf{u} \times \mathbf{v}||\), a result that is needed in Chapters 13 and 14.

11.5 Vector-Valued Functions and Curvilinear Motion

Theorems

Theorem A  (The limit of a vector-valued function is the vector of limits)
The theorem is proved in the Appendix.

Theorem B  Differentiation Formulas
The proof is given for part 4. Others are easy.

Homework  1, 3, 9, 13, 19–23, 33, 35, 44

Things to Remember  Derivatives and integrals of vector-valued functions. Velocity and acceleration.

11.6 Lines and Tangent Lines in Three-Space
Topics to Cover  Parametric and symmetric equations of a line in space. Tangent lines. Examples.

Theorems

None

Homework  1, 5–7, 9–14, 17–19, 23–25

Things to Remember  It is more difficult to get the equation of a line in three-space than in two-space. The direction vector for a tangent line is the derivative of the position function.

11.7 Curvature and Components of Acceleration
Topics to Cover  Curvature, radius of curvature. Examples. Alternative formulas for the curvature. Normal and tangential components of acceleration.

Theorems

Theorem A  (Formulas for the Curvature)
The proof is given.

Homework  1–2, 7–10, 15–17, 27–30

Things to Remember  Curvature is a relatively simple concept, but many problems are difficult because evaluating the integral is messy.

11.8 Surfaces in Three-Space
Topics to Cover  Cross sections, traces. Cylinders. Quadric surfaces. Examples.

Theorems

None

Homework  1–12, 21–23, 27–28

Things to Remember  The ability to visualize and draw graphs in three-space is essential to mastering Chapters 12–14.

11.9 Cylindrical and Spherical Coordinates
Topics to Cover  How to represent points in space by cylindrical or spherical coordinates. Conversion formulas. Graphs in cylindrical and spherical coordinates. Great circles.

Theorems

None

Homework  1–3, 7–10, 17–25

Things to Remember  Students should be comfortable working in these coordinate systems, which are used to evaluate double and triple integrals in Chapters 13 and 14.
12.1 Functions of Two or More Variables
Topics to Cover Definitions. 3-D plots. Contour plots. Level curves. Applications of contour plots. Functions of three or more variables.
Theorems None
Homework 1, 7-8, 17-18, 23, 25, 33, 39
Things to Remember The ability to visualize and manipulate functions of two (or more) variables is essential for the remainder of the book.

12.2 Partial Derivatives
Topics to Cover Partial derivatives as a rate of change. Examples. Geometric and physical interpretations. Higher-order partials.
Theorems None
Homework 1-8, 13, 15, 17-19, 25, 30, 49
Things to Remember Partial derivatives are used throughout the rest of the book.

12.3 Limits and Continuity
Topics to Cover Definition of limit of a function of two variables. Examples where limit does not exist. Continuity at a point and on a set.
Theorems
Theorem A (Limits for Polynomials and Rational Functions of Two Variables)
The proof is similar to the proof of Theorem 1.3B.
Theorem B Composition of Functions
The proof is similar to the proof of Theorem 1.6E.
Theorem C Equality of Mixed Partial derivatives
The proof is omitted. A counterexample for which the continuity of is lacking is given in Problem 42.
Homework 1-9, 17-22, 27-28, 35
Things to Remember Limits and continuity are trickier than for functions of one variable.

12.4 Differentiability
Topics to Cover Local linearity of a function of one variable. Local linearity of a function of two variables. Definition of differentiable as synonymous with locally linear. Tangent plane. Gradient. Properties of the gradient.
Theorems
Theorem A (Continuous Partial derivatives in a Neighborhood ⇒ Differentiable)
The proof is given.
Theorem B Properties of ∇
The proofs follow from the corresponding rules for partial derivatives. Part 3 is proved for the two-variable case.
Theorem C (Differentiability ⇒ Continuity)
The proof is given.

12.5 Directional Derivatives and Gradients
Topics to Cover Concept of rate of change in any direction. Definition of directional derivative. Computing the directional derivative. The gradient always points in the direction of most rapid increase in f. Higher dimensions.
Theorems
Theorem A (∂f/∂x(p) = u · ∇f(p))
The proof is given.
Theorem B (The gradient always points in the direction of most rapid increase in f)
The proof follows directly from Theorem A and from the rule u · v = |u| |v| cos θ.
Theorem C (The gradient is perpendicular to the level curve)
The proof is omitted.

12.6 The Chain Rule
Topics to Cover Two versions of the Chain Rule. Examples and applications.
Theorems
Theorem A (Chain Rule for ∂f/∂t(x(t), y(t)))
The proof is given.
Theorem B (Chain Rule for ∂f/∂s(x(s, t), y(s, t)), etc.)
The proof follows from Theorem A by holding one variable fixed.
Homework 1-4, 7-11, 15, 17, 20, 23
Things to Remember Remember the patterns in both versions of the Chain Rule.

12.7 Tangent Planes, Approximations
Topics to Cover Tangent plane to a surface defined by F(x, y, z) = k. Differentials and approximations. Second-order Taylor approximations.
Theorems
Theorem A Tangent Planes
The proof is given.
Homework 1-6, 9-11, 18, 29
Things to Remember The formula for the equation of the tangent line to a surface.
12.8 Maxima and Minima

Topics to Cover: Definitions, Existence of extrema, Examples, Second Partial Test.

Theorems
- **Theorem A** Max-Min Existence Theorem
  The proof is omitted.
- **Theorem B** Critical Point Theorem
  The proof is given.
- **Theorem C** Second Partial Test
  A sketch of the proof is given.

Things to Remember: The important skill in this section is translating a problem into mathematical terms so we can use the method of calculus to solve it.

12.9 The Method of Lagrange Multipliers

Topics to Cover: Geometric interpretation of the Lagrange multiplier, Examples, Two or more constraints.

Theorems
- **Theorem A** Lagrange's Method
  We forego a rigorous proof for the intuitive explanation given before the statement of the theorem.

Homework: 1-3, 9-11, 15

Things to Remember: Remember the geometric interpretation of the Lagrange multipliers.

Multiple Integrals

13.1 Double Integrals over Rectangles

Topics to Cover: Review the single Riemann integral, Definition of double integral, Integrability, Properties of the double integral, Linearity, additivity on rectangles, Comparison property, Evaluation of double integrals.

Theorems
- **Theorem A** Integrability Theorem
  The proof is difficult, so it is omitted.

Homework: 1, 5-7, 9, 15-17, 25

Things to Remember: Just like the single integral, the double integral is a limit of a sum.

13.2 Iterated Integrals

Topics to Cover: Slice, approximate, integrate to obtain the volume of a solid by an iterated integral, Evaluating iterated integrals using the Second Fundamental Theorem of Calculus, Examples of calculating volumes.

Theorems: None

Homework: 1-6, 9-10, 13, 17-19, 21, 29, 31

Things to Remember: A double integral can be evaluated by writing it as an iterated integral. Students will need to evaluate iterated integrals in the rest of Chapter 13.

13.3 Double Integrals over Nonrectangular Regions

Topics to Cover: x-simple and y-simple sets, Definition of double integral over nonrectangular region, Evaluating iterated integrals over nonrectangular regions.

Theorems: None

Homework: 1-8, 11, 15, 17, 21-24, 33, 37

Things to Remember: Choosing the limits is the hardest part of evaluating iterated integrals. Fix one variable and integrate over the other.

13.4 Double Integrals in Polar Coordinates

Topics to Cover: Polar rectangles, \( \frac{dx \, dy}{r \, dr \, d\theta} \), r-simple and \( \theta \)-simple regions, Volume of a solid.

Theorems: None

Homework: 1-8, 11, 13, 16

Things to Remember: The formulas for the center of mass are generalizations of those given in Chapter 5.

13.5 Applications of Double Integrals

Topics to Cover: Mass, center of mass, centroid, Moment of inertia, radius of gyration (optional), Examples.

Theorems: None

Homework: 1-4, 7-9, 11-13, 15-18, 21, 23

Things to Remember: The formulas for the center of mass are generalizations of those given in Chapter 5.

13.6 Surface Area

Topics to Cover: Review area of a parallelogram as \( |a \times v| \), Slice (the region \( S \)), approximate (the surface area above each small rectangle), and integrate (to form a double integral). \( A(G) = \int_S \sqrt{f_1^2 + f_2^2 + 1} \, dA \).

Examples.

Theorems: None

Homework: 1-6, 11, 13, 17-19, 24

Things to Remember: Formula for surface area. A similar development will occur in Chapter 14 when we study surface integrals.

13.7 Triple Integrals (Cartesian Coordinates)

Topics to Cover: Riemann sum for function of three variables, Definition of triple integral over rectangular box and over a general region, x-simple, y-simple, and z-simple regions, Examples, Center of mass.
13.8 Triple Integrals (Cylindrical and Spherical Coordinates)

Topics to Cover
- Review cylindrical and spherical coordinates.
- Setting up integrals in these coordinate systems.

Theorems
- None

Homework
- 1-7, 15, 17, 21, 25, 27, 29, 33

Things to Remember
- Emphasize that setting up the limits is usually the hardest part. It is often beneficial to set up a number of iterated integrals and leave it to the students to work them out.

13.9 Change of Variables in Multiple Integrals

Topics to Cover
- Functions from $\mathbb{R}^2$ to $\mathbb{R}^2$, $u$- and $v$-curves.
- Jacobians: Change of Variable Theorem, Making a change of variable in a double integral, Change of variables to cylindrical coordinates.

Theorems
- Theorem A Change of Variables for Multiple Integrals
- The proof is given. A rigorous proof would require Green’s Theorem.

Homework
- 1-8, 11, 13, 15-18, 20, 27

Things to Remember
- Formulas for $dV$.
- Emphasize that $dV$ is not simply $dp\,dq\,dr$.

Vector Calculus

14.1 Vector Fields

Topics to Cover

Theorems
- None

Homework
- 1, 4, 7, 13, 17-18, 21

Things to Remember
- A working knowledge of vector fields is needed throughout the rest of Chapter 14.

14.2 Line Integrals

Topics to Cover
- Parametric curves, parametrizations, orientation.
- Line integrals of the form $\int_C f(x, y)\, ds$,
- $\int_C M\, dx + N\, dy$,
- $\int_C M\, dx + N\, dy + P\, dz$.

Examples and applications.

Theorems
- None

Homework
- 1-4, 7, 9, 15-16, 19-21, 26, 32

Things to Remember
- Notation is the key. Line integrals can be written in a number of different ways, so be sure students understand notation.

14.3 Independence of Path

Topics to Cover
- Fundamental Theorem of Line Integrals.
- (Emphasize that to evaluate $\int_F \cdot dr$, you must be able to find a potential function for $F$.) Independence of Path Theorem. Recovering a function from its gradient. Conservation of energy (optional).

Theorems
- Theorem A Fundamental Theorem of Line Integrals
  - The proof is given.
- Theorem B Independence of Path Theorem
  - The proof is given.
- Theorem C Equivalent Conditions for Line Integrals
  - The proof is given.
- Theorem D ($F$ is Conservative $\iff$ curl $F = 0$)
  - The “if” follows from Green’s Theorem in the two-variable case and Stokes’s Theorem in the three-variable case. The “only if” part is easy.

Homework
- 1-5, 11, 13, 17, 22

Things to Remember
- $F$ must be conservative to apply the Fundamental Theorem of Line Integrals to evaluate $\int_F \cdot dr$. If you know that $F$ is conservative, you can find $f$ by the method described in the text.

14.4 Green’s Theorem in the Plane

Topics to Cover
- Statement and proof of Green’s Theorem (the proof is not difficult). Examples and applications. Vector forms of Green’s Theorem $\oint_C F \cdot n\, ds = \iint_D \text{div} F\, dA$, and $\oint_C F \cdot T\, ds = \iint_D (\text{curl} F) \cdot k\, dA$. Flux in the plane.

Theorems
- Theorem A Green’s Theorem
  - The proof is given.

Homework
- 1-3, 7, 9-11, 13, 15, 18

Things to Remember
- Green’s Theorem relates a double integral over a region and a line integral along its boundary.
- If you emphasize the vector forms of Green’s Theorem, it
will make Gauss's Theorem (Section 14.6) and Stokes's Theorem (Section 14.7) easier to understand. Warn students to use Green's Theorem only for a closed curve, and the Independence of Path Theorem (along with the FTLI) only for conservative fields.

14.5 Surface Integrals

**Topics to Cover** Emphasize that \( f(x, y, z) \) is defined only on the surface \( z = f(x, y) \). Slice, approximate, integrate. Formula for surface integral. Examples. Flux across a surface. Formula for flux.

**Theorems**

- **Theorem A** (Formula for Surface Integral):
  \[
  \iint_S g(x, y, z) \, dS = \iiint_\Omega g(x, y, f(x, y)) \sqrt{f_x'^2 + f_y'^2 + 1} \, dx \, dy
  \]
  The derivation (slice, approximate, integrate) is given before the theorem.

- **Theorem B** (flux \( F = \oint_C F \cdot T \, ds \))
  \[
  \iint_S (-Mf_x - Nf_y + P) \, dy \, dx
  \]
  The proof is given.

**Homework** 1–3, 8–11, 13, 16–17, 23, 25

**Things to Remember** Remember the concept of the surface integral and the formula for evaluating a surface integral, and how to obtain flux with a surface integral.

14.6 Gauss's Divergence Theorem


**Theorems**

- **Theorem A** Gauss's Divergence Theorem
  \[
  \iiiint_V F \cdot n \, dV = \iiint_S \text{div } F \, dS
  \]
  The proof is long but not difficult.

**Homework** 1–3, 7, 9, 15, 18

**Things to Remember** Gauss's Theorem relates a triple integral over a solid and a surface integral over its boundary. Flux through the boundary of a solid can be obtained by evaluating a surface integral over the solid's boundary or by evaluating a triple integral over the solid.

14.7 Stokes's Theorem


**Theorems**

- **Theorem A** Stokes's Theorem
  \[
  \oint_C F \cdot n \, ds = \iint_S (\text{curl } F) \cdot n \, dS
  \]
  The proof is omitted.

**Homework** 1–2, 5, 7, 9, 15

**Things to Remember** Stokes's Theorem relates a surface integral and a line integral along the edge of the surface.