CALCULUS 1210-002

Quiz 5

Your Name (write it legibly): ___________________________ ID: ___________________________

INSTRUCTIONS

Show Your Work; No Calculators. No Phones and other Electronic Equipment. Box your answers. No two solutions

1. Find the derivative of $y$:

(a) $y = x^3 + x^2 + \sqrt{x} + 1$

\[
y' = 3x^2 + 2x + \frac{1}{2\sqrt{x}}
\]

(b) $y = \frac{x+1}{x^2+1}$

\[
y' = \left(\frac{x+1}{x^2+1}\right)' = \frac{(x+1)'(x^2+1)-(x+1)(x^2+1)'}{(x^2+1)^2} = \frac{x^2+1 - (x+1)2x}{(x^2+1)^2} = \frac{x^2+1 - 2x^2 - 2x}{(x^2+1)^2} = \frac{-x^2 - 2x + 1}{(x^2+1)^2}
\]

(c) $y = \sin x \cos x$

\[
y' = (\sin x \cos x)' = \cos (\sin x)' \cos x + \sin x (\cos x)' = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x
\]

2. Find the derivative of $y$:

(a) $y = \sqrt{x^2 + 1}$

\[
y' = \frac{1}{2\sqrt{x^2+1}} \cdot x = \frac{x}{\sqrt{x^2+1}}
\]

(b) $y = \sin (\sqrt{x})$

\[
y' = \cos (\sqrt{x}) (\sqrt{x})' = \frac{\cos (\sqrt{x})}{2\sqrt{x}}
\]
3. Let \( y = x^3 + x^2 + \frac{1}{x} \). Then

(a) \( y' = 3x^2 + 2x - \frac{1}{x^2} \)

(b) \( y'' = 6x + 2 + \frac{2}{x^3} \)

(c) \( y''' = 6 - \frac{6}{x^4} \)

4. Find an equation of a line tangent to \( y = \frac{1}{x^2} \) at \( x = 2 \)

\[
y' = -\frac{2}{x^3} + 1
\]

\( y'(2) = -\frac{2}{2^3} = -\frac{1}{4} + 1 \) ← slope

Point: \( (2, \frac{1}{4}) \)

Equation

\[
y - \frac{1}{4} = -\frac{1}{4} (x - 2) + 1
\]

\[
y - \frac{1}{4} = -\frac{1}{4} x + \frac{1}{2}
\]

\[
y = -\frac{1}{4} x + \frac{3}{4}
\]