try $y = e^{rx}$, $L(y) = (r^2+2r-15)e^{rx} = 0$

$(r+5)(r-3) = 0$, $r = 3, -5$

$$\boxed{y(x) = c_1 e^{3x} + c_2 e^{-5x}}$$

35. $y'' + 5y' = 0$ try $y = e^{rx}$ $L(y) = (r^2 + 5r)e^{rx} = 0$

$r(r+5) = 0$

$r = 0, -5$

$$\boxed{y(x) = c_1 + c_2 e^{-5x}}$$

39. $4y'' + 4y' + y = 0$ for $y = e^{rx}$; $L(y) = (4r^2 + 4r + 1)e^{rx} = 0$

$r = \frac{-4 \pm \sqrt{16-16}}{8} = -\frac{1}{2}$

(oh; $(2r+1)^2 = 0$

$$\boxed{y(x) = c_1 e^{-\frac{1}{2}x} + c_2 x e^{-\frac{1}{2}x}}$$

5.2. 2) $c_1(5) + c_2(2-3x^2) + c_3(10+15x^2) = 0$

$1(5c_1 + 2c_2 + 10c_3)$

$+ x^2(-3c_2 + 15c_3) \quad = 0$

$$\left[\begin{array}{ccc|c} 5 & 2 & 10 & 0 \\ 0 & -3 & 15 & 0 \end{array}\right]$$

$$\begin{array}{r} -2R_2+R_1 \\ -R_2/3 \end{array} \left[\begin{array}{ccc|c} 5 & 0 & 20 & 0 \\ 0 & 1 & -5 & 0 \end{array}\right]$$

$$\begin{array}{r} R_1/4 \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & -5 & 0 \end{array}\right]$$

$c_3 = t$

$c_2 = 5t$

$c_1 = -4t$

$\vec{c} = t\begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix}$ e.g. $t = 1$: $\begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix}$

∇ so $-4\overset{f}{[5]} + 5\overset{g}{[2-3x^2]} + 1\overset{h}{[10+15x^2]}$

$= 0$

So we only need to get it non-zero for at least 1 x-value in the interval.

$$W = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix} = \begin{vmatrix} e^x & \cos x & \sin x \\ e^x & -\sin x & \cos x \\ e^x & -\cos x & -\sin x \end{vmatrix}$$

at $x = 0$ this is $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 1 + 1 = 2$ so $\underline{l.i.}$

11) $$W = \begin{vmatrix} x & xe^x & x^2e^x \\ 1 & e^x(x+1) & e^x(x^2+2x) \\ 0 & e^x(x+2) & e^x(x^2+2x+2x+2) \end{vmatrix}$$

at $x = 0$ get $\begin{vmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 0 \end{vmatrix} = 0$,

so try another x : say $x = 1$:

$$\begin{vmatrix} 1 & e & e \\ 1 & 2e & 3e \\ 0 & 3e & 7e \end{vmatrix} = e^2 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 3 & 7 \end{vmatrix}$$ (factored an e out of col$_3$ & col$_4$

$= e^2[14+3-9-7] = e^2 \neq 0$

13) $y(x) = c_1e^x + c_2e^{-x} + c_3e^{-2x}$

$y' = c_1e^x - c_2e^{-x} - 2c_3e^{-2x}$

$y'' = c_1e^x + c_2e^{-x} + 4c_3e^{-2x}$

$y(0) = c_1 + c_2 + c_3 = 1$

$y'(0) = c_1 - c_2 - 2c_3 = 0$

$y''(0) = c_1 + c_2 + 4c_3 = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -1 & -2 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ -R_1+R_2 \\ -R_1+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -3 & -1 \\ 0 & 0 & 3 & -1 \end{array}\right]$$

$$\begin{array}{r} -R_3+R_1 \\ 3R_3+R_2 \\ R_3/3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 4/3 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & -1/3 \end{array}\right]$$

$$\begin{array}{r} R_1-R_2 \\ -R_2/2 \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1/3 \end{array}\right]$$

$c_1 = 1/3$

$c_2 = 1$

$c_3 = -1/3$

$$\boxed{y(x) = \frac{1}{3}e^x + e^{-x} - \frac{1}{3}e^{-2x}}$$

5.2 22) $y(x) = -3 + c_1e^{2x} + c_2e^{-2x}$

$y'(x) = 2c_1e^{2x} - 2c_2e^{-2x}$

$y(0) = -3 + c_1 + c_2 = 0$

$y'(0) = 2c_1 - 2c_2 \quad = 10$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & -2 & 10 \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & -1 & 5 \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -2 & 2 \end{array}\right] -R_1+R_2 \quad \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -1 \end{array}\right]$$

$c_1 = 4$
$c_2 = -1$

$$\boxed{y(x) = -3 + 4e^{2x} - e^{-2x}}$$

26) $y'' + 2y = 4$ $\qquad$ $y'' + 2y = 6x$

$y_p = 2$ $\qquad$ $y_p = 3x$

so $\boxed{y_p = 2 + 3x}$ solves $y'' + 2y = 4 + 6x$

5.3 3) $y'' + 3y' - 10y = 0$

for $y = e^{rx}$ need $r^2 + 3r - 10 = 0$

$(r+5)(r-2) = 0$; $r = -5, 2$

$$\boxed{y(x) = c_1e^{2x} + c_2e^{-5x}}$$

10) $5y^{(4)} + 3y^{(3)} = 0$

for $y = e^{rx}$ get $5r^4 + 3r^3 = 0$

$r^3[5r + 3] = 0$

$r = 0$ (multiplicity 3)
$r = -3/5$

$$\boxed{y(x) = c_1 + c_2x + c_3x^2 + c_4e^{-3/5x}}$$

14) $y^{(4)} + 3y'' - 4y = 0$

for $y = e^{rx}$ get $r^4 + 3r^2 - 4 = 0$

$(r^2+4)(r^2-1) = 0$; $r = \pm 2i, \pm i$

$e^{(2i)x} = \cos 2x + i\sin 2x$

$e^{ix} = \cos x + i \sin x$

$$\boxed{y(x) = c_1\cos 2x + c_2\sin 2x + c_3\cos x + c_4\sin x}$$

22) $9y'' + 6y' + 4y = 0$

$9r^2 + 6r + 4 = 0$

$$r = \frac{-6 \pm \sqrt{36-144}}{18} = \frac{-6 \pm \sqrt{-108}}{18} = \frac{-6 \pm 6\sqrt{3}\,i}{18}$$

$$= -\frac{1}{3} \pm \frac{1}{\sqrt{3}}i$$

$$e^{(-\frac{1}{3} + \frac{1}{\sqrt{3}}i)x} = e^{-\frac{1}{3}x}\left[\cos\frac{x}{\sqrt{3}} + i\sin\frac{x}{\sqrt{3}}\right]$$

$$y(x) = e^{-\frac{1}{3}x}\left[c_1\cos\frac{x}{\sqrt{3}} + c_2\sin\frac{x}{\sqrt{3}}\right]$$

product rule

$$y'(x) = e^{-\frac{1}{3}x}\left[-\frac{1}{3}\left(c_1\cos\frac{x}{\sqrt{3}} + c_2\sin\frac{x}{\sqrt{3}}\right) + -\frac{c_1}{\sqrt{3}}\sin\frac{x}{\sqrt{3}} + \frac{c_2}{\sqrt{3}}\cos\frac{x}{\sqrt{3}}\right]$$

$y(0) = c_1 \quad = 3$ $\qquad$ $c_1 = 3$

$y'(0) = -\frac{1}{3}c_1 + \frac{1}{\sqrt{3}}c_2 = 14$ $\qquad$ $-1 + \frac{c_2}{\sqrt{3}} = 14$

$\frac{c_2}{\sqrt{3}} = 15$, $\quad c_2 = 15\sqrt{3}$

$$\boxed{y(x) = e^{-\frac{1}{3}x}\left[3\cos\frac{x}{\sqrt{3}} + 15\sqrt{3}\sin\frac{x}{\sqrt{3}}\right]}$$

29) $y^{(3)} + 27y = 0$

$r^3 + 27 = 0$ $\qquad$ $r = -3$ is a root ; $r+3$ is a factor

$$\begin{array}{r|l} & r^2 - 3r + 9 \\ r+3 & r^3 + \quad\quad 27 \\ & r^3 + 3r^2 \\ \hline & -3r^2 \quad + 27 \\ & -3r^2 - 9r \\ \hline & 9r + 27 \\ & 9r + 27 \\ \hline & 0 \end{array}$$

roots $r = \frac{3 \pm \sqrt{9-36}}{2}$

$= \frac{3 \pm \sqrt{-27}}{2}$

$= \frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$

$$e^{(\frac{3}{2} + \frac{3\sqrt{3}}{2}i)x} = e^{\frac{3}{2}x}\left[\cos\frac{3\sqrt{3}}{2}x + i\sin\frac{3\sqrt{3}}{2}x\right]$$

$$\boxed{y(x) = e^{\frac{3}{2}x}\left[c_1\cos\frac{3\sqrt{3}}{2}x + c_2\sin\frac{3\sqrt{3}}{2}x\right] + c_3e^{-3x}}$$

⑦

37) $y^{(4)} - y^{(3)} = 0$

characteristic poly $r^4 - r^3 = 0$

$r^3(r-1) = 0$ $e^x, 1, x, x^2$

$$\begin{aligned} y(x) &= c_1 + c_2x + c_3x^2 + c_4e^x \\ y' &= c_2 + 2c_3x + c_4e^x \\ y'' &= 2c_3 + c_4e^x \\ y^{(4)} &= c_4e^x \end{aligned}$$

$$\begin{aligned} y(0) &= c_1 + c_4 = 18 && c_1 = 11 \\ y'(0) &= c_2 + c_4 = 12 && c_2 = 5 \\ y''(0) &= 2c_3 + c_4 = 13 && c_3 = 3 \\ y'''(0) &= c_4 = 7 && c_4 = 7 \end{aligned}$$

$$\boxed{y(x) = 11 + 5x + 3x^2 + 7e^x}$$

33) $y^{(3)} + 3y'' - 54y = 0$

$r^3 + 3r^2 - 54 = 0$ since $y = e^{3x}$ is soln $r = 3$ is root, $r-3$ is factor

$$\begin{array}{r} r^2 + 6r + 18 \\ r-3 \overline{\smash{)}\, r^3 + 3r^2 - 54} \\ \underline{r^3 - 3r^2} \\ 6r^2 \\ \underline{6r^2 - 18r} \\ 18r - 54 \\ \underline{18r - 54} \\ 0 \end{array}$$

$$r = \frac{-6 \pm \sqrt{36-72}}{2} = \frac{-6 \pm 6i}{2} = -3 \pm 3i$$

$e^{-3x}(\cos 3x + i\sin 3x)$

$$\boxed{y(x) = c_1e^{3x} + e^{-3x}\left[c_2\cos 3x + c_3\sin 3x\right]}$$

Math 2250-4

HW due Wed 10/31

5.4 (4, 5, 7), 10, 12, (13, 18, 23)

5.5 ③ 4, 13, ⑫ ⑲ (34, 37, 43, 49) 50, (52)

4. $m = .25$ kg

$k(.25) = 9$ N

$k = 36$ N/m

$x_0 = 1$ m

$V_0 = -5$ m/s

$$.25\,x'' + 36\,x = 0$$

$$x'' + 144x = 0$$

$$x = e^{rt} \cdot \quad p(r) = r^2 + 144 = 0$$

$$r^2 = -144$$

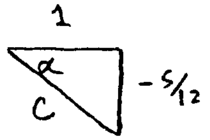
$$r = \pm 12i$$

$$e^{rt} = e^{\pm 12it} = \cos 12t \pm i \sin 12t$$

so $$x_H(t) = A\cos 12t + B\sin 12t$$

$$x(0) = 1 = A \qquad A = 1$$

$$x'(0) = -5 = 12B; \qquad B = -5/12$$



$$C = \sqrt{1 + \frac{25}{144}} = \sqrt{\frac{169}{144}} = \boxed{\frac{13}{12} \leftarrow \text{amplitude}}$$

$$\alpha = \tan^{-1}\left(-\frac{5}{12}\right) \leftarrow \text{phase}$$

$$\boxed{x(t) = \frac{13}{12}\cos(12t - \alpha) \cong 1.083\cos(12t + .430)}$$

$$\boxed{\text{period} = \frac{2\pi}{12} = \frac{\pi}{6} \text{ seconds}}$$

5. Pendulum eqtn is

$$\theta''(t) + \frac{g}{L}\theta = 0$$

so $\omega_0 = \sqrt{\frac{g}{\ell}}$ rad/sec.,

$$\theta(t) = C\cos(\omega_0 t - \alpha)$$

$$\text{period} = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{\ell}{g}}$$

in this problem

$$\frac{P_1}{P_2} = \frac{2\pi\sqrt{\frac{L_1}{g_1}}}{2\pi\sqrt{\frac{L_2}{g_2}}} = \sqrt{\frac{L_1}{L_2}}\sqrt{\frac{g_2}{g_1}} = \sqrt{\frac{L_1}{L_2}}\sqrt{\frac{GM/R_2^2}{GM/R_1^2}}$$

$$= \sqrt{\frac{L_1}{L_2}}\,\frac{R_1}{R_2} \quad \checkmark$$

7. $$\frac{P_1}{P_2} = \frac{R_1}{R_2}\sqrt{\frac{L_1}{L_2}}$$ unit don't matter (cancel in quotient)

$P_1 = P_2$

$$\Rightarrow \frac{R_2}{R_1} = \sqrt{\frac{L_1}{L_2}}$$

$R_1 = 3960$

$L_1 = 100.1$

$L_2 = 100.0$

$$R_2 = 3960\sqrt{\frac{100.1}{100.0}}$$

$$\approx 3961.98 \text{ miles}$$

$$\begin{array}{r} 3961.979 \\ -\,3960.00 \\ \hline 1.979 \text{ miles} \end{array}$$

$\times 5280$ ft/m

$$\boxed{\approx 10{,}452 \text{ ft}}$$

⑬ $mx''+cx'+kx=0$

$$\begin{cases}10x''+9x'+2x=0\\ x(0)=0\\ x'(0)=5\end{cases}$$

$x=e^{rt}:\ 10r^2+9r+2=0$

$(5r+2)(2r+1)=0$

$r=-1/2,\ -2/5$ (overdamped)

$x(t)=c_1e^{-1/2\,t}+c_2e^{-2/5\,t}$

$*\quad x'(t)=-\frac{1}{2}c_1e^{-1/2\,t}-\frac{2}{5}c_2e^{-2/5\,t}$

$x(0)=0=c_1+c_2$

$x'(0)=5=-\frac{1}{2}c_1-\frac{2}{3}c_2$

$c_1+c_2=0\quad)5$

$-5c_1-4c_2=50\quad)1$

$$\boxed{\begin{matrix}c_2=50\\ c_1=-50\end{matrix}}$$

$x(t)=-50e^{-1/2\,t}+50e^{-2/5\,t}$

$$\boxed{x(t)=50e^{-.4t}\left[1-e^{-.1t}\right]}$$

$x(t)>0\quad t>0$

x_{max} when $x'(t)=0$

$*\quad x'(t)=+25e^{-1/2\,t}-20e^{-2/5\,t}$

$=e^{-.4t}\left[-20+25e^{-.1t}\right]=0:\quad 25e^{-.1t}=20$

$e^{-.1t}=.8$

$-.1t=\ln(.8)$

$T=\frac{\ln .8}{-.1}\approx 2.23$

$x_{max}=x(T)$

where $25e^{-.1T}=20$; $e^{.1T}=\frac{5}{4}$

so $x(T)=50\left(\frac{4}{5}\right)^4\left(1-\frac{4}{5}\right)$

$=\frac{50\cdot 4^4}{5^5}=10\left(\frac{4}{5}\right)^4\approx\boxed{4.096}$

⑱

$$\begin{cases}2x''+12x'+50x=0\\ x(0)=0\\ v(0)=-8\end{cases}$$

$x''+6x'+25x=0$

$r^2+6r+25=0$

$(r+3)^2+16=0$

$(r+3-4i)(r+3+4i)=0\qquad r=-3\pm4i$

underdamped

$e^{(-3+4i)t}=e^{-3t}(\cos 4t+i\sin 4t)$

$x_H(t)=e^{-3t}\left(A\cos 4t+B\sin 4t\right)$

$x_H(0)=0=A$

$x_H'(0)=-8=-3A+4B;\quad B=-2$

$$\boxed{x_H(t)=-2e^{-3t}\sin 4t}$$

requested form ↓↓

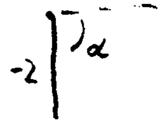
$=e^{-3t}(-2\sin 4t)=\boxed{2e^{-3t}\cos(4t+\pi/2)}$

"triangle" $A=0$

$B=-2$

$C=2$

$\alpha=-\pi/2$



§5.4 ㉓ a) $100x'' + kx = 0$

$$w_0 = \sqrt{\frac{k}{100}}$$

told $f = \frac{w_0}{2\pi} = 80$ cycles/min $= \frac{4}{3}$ cycles/sec.

$$w_0 = \frac{8\pi}{3} \text{ rad/sec}$$

$$\left(\frac{8\pi}{3}\right)^2 = w_0^2 = \frac{k}{100} \ ; \qquad k = 100\left(\frac{8\pi}{3}\right)^2 \approx 7018 \text{ lb/ft}$$

23 b) $\frac{w_1}{2\pi} = \frac{78}{60}$ cycles/sec

$= 1.3$ cycles/sec

$w_1 = 2.6\pi$ rad/sec.

$x'' + cx' + 70.18 = 0$ (divided eqtn by 100)

$r^2 + cr + 70.18 = 0$

$$r = \frac{-c \pm \sqrt{c^2 - 280.74}}{2}$$

$$2.6\pi = w_1 = \frac{\sqrt{280.74 - c^2}}{2}$$

$$4(2.6\pi)^2 = 280.74 - c^2$$

$$c^2 = 280.74 - 4(2.6\pi)^2$$

$$c \approx 3.724$$

$$p = \frac{c}{2} \approx 1.862$$

$$x(t) = e^{-1.862t}\left(C\cos(2.6\pi t - \alpha)\right)$$

solve $e^{-1.862t} = .01$

$$t = \frac{\ln(.01)}{-1.862} \approx \boxed{2.47 \text{ sec}}$$

§5.5 3) $y'' - y' - 6y = 2\sin 3x$

$-6($ $y_p = A\cos 3x + B\sin 3x$

$-1($ $y_p' = -3A\sin 3x + 3B\cos 3x$

$1($ $y_p'' = -9A\cos 3x - 9B\sin 3x$

$$L(y_p) = \cos 3x\left[-9A - 3B - 6A\right] + \sin 3x\left[-9B + 3A - 6B\right]$$

$-5A - B = 0$

$-15A - 3B = 0$

$3A - 15B = 2$

$$A = \frac{\begin{vmatrix} 0 & -1 \\ 2 & -15 \end{vmatrix}}{\begin{vmatrix} -5 & -1 \\ 3 & -15 \end{vmatrix}} = \frac{2}{78} = \frac{1}{39} \ ; \quad B = -\frac{5}{39}$$

$$\boxed{y_p = \frac{1}{39}\cos 3x - \frac{5}{39}\sin 3x}$$