Math 2250-3

HW due 10/22

5.1 1, 2, 4, 5, 11, 13, 17, 27 29, 30, 31, 33, 34, 35, 39

5.2 2 5 9, 11, 13, 21, 22 25 26

5.1 2) $y''-9y=0$

$y_1=e^{3x}$, $y_1''=9e^{3x}$, $y_1''-9y_1=9e^{3x}-9e^{3x}=0$

$y_2=e^{-3x}$, $y_2''=9e^{3x}$, so $y_2''-9y_2=9e^{3x}-9e^{3x}=0$

(or for $y=e^{rx}$, $L(y)=(r^2-9)e^{rx}$ roots $r=\pm 3$ ✓)

$y(x)=c_1e^{3x}+c_2e^{-3x}$

$y'(x)=3c_1e^{3x}-3c_2e^{-3x}$

$y(0)=c_1+c_2=-1$

$y'(0)=3c_1-3c_2=15$

$c_1=\dfrac{\begin{vmatrix}-1 & 1\\ 15 & -3\end{vmatrix}}{\begin{vmatrix}1 & 1\\ 3 & -3\end{vmatrix}}=\dfrac{-12}{-6}=2$

$c_2=\dfrac{\begin{vmatrix}1 & -1\\ 3 & 15\end{vmatrix}}{-6}=\dfrac{18}{-6}=-3$

$\boxed{y(x)=2e^{3x}-3e^{-3x}}$

4) $y''+25y=0$

(characteristic poly $r^2+25=0$; $r=\pm 5i$)

$y_1=\cos 5x$, $y_1''=-25\cos 5x$, $y_1''+25y_1=-25\cos 5x+25\cos 5x=0$

$y_2=\sin 5x$, $y_2''=-25\sin 5x$, so $L(y)=-25\sin 5x+25\sin 5x=0$

$y(x)=c_1\cos 5x+c_2\sin 5x$

$y'(x)=-5c_1\sin 5x+5c_2\cos 5x$

$y(0)=c_1=10$

$y'(0)=5c_2=-10$

$c_1=10$, $c_2=-2$

$\boxed{y(x)=10\cos 5x-2\sin 5x}$

5) $y''-3y'+2y=0$

(char poly $r^2-3r+2=(r-2)(r-1)$)

$y_1=e^x$ $L(y_1)=e^x-3e^x+2e^x=0$

$y_2=e^{2x}$ $L(y_2)=4e^{2x}-3(2e^{2x})+2e^{2x}=0$

$y(x)=c_1e^x+c_2e^{2x}$

$y'(x)=c_1e^x+2c_2e^{2x}$

$y(0)=c_1+c_2=1$

$y'(0)=c_1+2c_2=0$

$c_2=-1$, $c_1=2$

$\boxed{y(x)=2e^x-e^{-2x}}$

(char poly $r^2-2r+2=0$; $r=\frac{2\pm\sqrt{4-8}}{2}=1\pm i$)

11) $y''-2y'+2y=0$

$y_1=e^x\cos x$

$y_1'=e^x(\cos x-\sin x)$

$y_1''=e^x(\cos x-\sin x-\sin x-\cos x)=e^x(-2\sin x)$

$L(y_1)=e^x[-2\sin x-2(\cos x-\sin x)+2\cos x]=0$ ✓

$y_2=e^x\sin x$

$y_2'=e^x[\sin x+\cos x]$

$y_2''=e^x[\sin x+\cos x+\cos x-\sin x]=e^x[2\cos x]$

$L(y_2)=e^x[2\cos x-2[\sin x+\cos x]+2\sin x]=0$ ✓

$y=c_1e^x\cos x+c_2e^x\sin x$

$y'=c_1e^x(\cos x-\sin x)+c_2e^x(\sin x+\cos x)$

$y(0)=c_1=0$

$y'(0)=c_1+c_2=5$

$c_1=0$, $c_2=5$

$\boxed{y(x)=5e^x\sin x}$

13) $L(y)=x^2y''-2xy'+2y=0$

$y_1=x$, $y_1'=1$, $y_1''=0$: $L(y)=x^2(0)-2x(1)+2x=0$

$y_2=x^2$, $y_2'=2x$, $y_2''=2$: $L(y_2)=x^2(2)-2x(2x)+2x^2=0$

$y(x)=c_1x+c_2x^2$

$y'(x)=c_1+2c_2x$

$y(1)=c_1+c_2=3$

$y'(1)=c_1+2c_2=1$

$c_2=-2$, $c_1=5$

$\boxed{y(x)=5x-2x^2}$

17) $y=1/x$; $y'=-1/x^2$; $L(y)=y'+y^2=-\frac{1}{x^2}+\left(\frac{1}{x}\right)^2=0$ ✓

if $y=c/x$, $y'=-c/x^2$ so $L(y)=-c/x^2+\frac{c^2}{x^2}=\frac{1}{x^2}[c^2-c]=\frac{1}{x^2}c[c-1]$

$=0$ only when $c=0$ or 1.

5.1 34 $y'' + 2y' - 15y = 0$

try $y = e^{rx}$, $L(y) = (r^2 + 2r - 15)e^{rx} = 0$

$(r+5)(r-3) = 0$, $r = 3, -5$

$$\boxed{y(x) = c_1 e^{3x} + c_2 e^{-5x}}$$

35. $y'' + 5y' = 0$ try $y = e^{rx}$ $L(y) = (r^2 + 5r)e^{rx} = 0$

$r(r+5) = 0$

$r = 0, -5$

$$\boxed{y(x) = c_1 + c_2 e^{-5x}}$$

39. $4y'' + 4y' + y = 0$ try $y = e^{rx}$; $L(y) = (4r^2 + 4r + 1)e^{rx} = 0$

$r = \dfrac{-4 \pm \sqrt{16-16}}{8} = -\frac{1}{2}$

(oh; $(2r+1)^2 = 0$

$$\boxed{y(x) = c_1 e^{-\frac{1}{2}x} + c_2 x e^{-\frac{1}{2}x}}$$

5.2 2) $c_1(5) + c_2(2 - 3x^2) + c_3(10 + 15x^2) = 0$

$1(5c_1 + 2c_2 + 10c_3)$

$+ x^2(-3c_2 + 15c_3) = 0$

$$\left[\begin{array}{ccc|c} 5 & 2 & 10 & 0 \\ 0 & -3 & 15 & 0 \end{array}\right]$$

$$\begin{array}{l} -2R_2 + R_1 \\ -R_2/3 \end{array} \left[\begin{array}{ccc|c} 5 & 0 & 20 & 0 \\ 0 & 1 & -5 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1/4 \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & -5 & 0 \end{array}\right]$$

$c_3 = t$

$c_2 = 5t$

$c_1 = -4t$

$\vec{c} = t\begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix}$ e.g. $t = 1$: $\begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix}$

$$\begin{array}{cccc} & f & g & h \end{array}$$

so $-4[5] + 5[2 - 3x^2] + 1[10 + 15x^2] = 0$

5.2 9) dependent would mean Wronskian is zero for all x-values. So we only need to get it non-zero for at least 1 x-value in the interval.

$$W = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix} = \begin{vmatrix} e^x & \cos x & \sin x \\ e^x & -\sin x & \cos x \\ e^x & -\cos x & -\sin x \end{vmatrix}$$

at $x = 0$ this is $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 1 + 1 = 2$ so l.i.

11) $$W = \begin{vmatrix} x & xe^x & x^2e^x \\ 1 & e^x(x+1) & e^x(x^2+2x) \\ 0 & e^x(x+2) & e^x(x^2+2x+2x+2) \end{vmatrix}$$

at $x = 0$ get $\begin{vmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 0 \end{vmatrix} = 0$,

so try another x : say $x = 1$:

$$\begin{vmatrix} 1 & e & e \\ 1 & 2e & 3e \\ 0 & 3e & 7e \end{vmatrix} = e^2 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 3 & 7 \end{vmatrix}$$

(factored an e out of col$_3$ & col$_4$)

$= e^2[14 + 3 - 9 - 7] = e^2 \neq 0$

13) $y(x) = c_1e^x + c_2e^{-x} + c_3e^{-2x}$

$y' = c_1e^x - c_2e^{-x} - 2c_3e^{-2x}$

$y'' = c_1e^x + c_2e^{-x} + 4c_3e^{-2x}$

$y(0) = c_1 + c_2 + c_3 = 1$

$y'(0) = c_1 - c_2 - 2c_3 = 0$

$y''(0) = c_1 + c_2 + 4c_3 = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -1 & -2 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ -R_1 + R_2 \\ -R_1 + R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -3 & -1 \\ 0 & 0 & 3 & -1 \end{array}\right]$$

$$\begin{array}{l} -R_3 + R_1 \\ 3R_3 + R_2 \\ R_3/3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 4/3 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & -1/3 \end{array}\right]$$

$$\begin{array}{l} R_1 - R_2 \\ -R_2/2 \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1/3 \end{array}\right]$$

$c_1 = 1/3$

$c_2 = 1$

$c_3 = -1/3$

$$\boxed{y(x) = \frac{1}{3}e^x + e^{-x} - \frac{1}{3}e^{-2x}}$$

5.2 22) $y(x) = -3 + c_1 e^{2x} + c_2 e^{-2x}$

$y'(x) = 2c_1 e^{2x} - 2c_2 e^{-2x}$

$y(0) = -3 + c_1 + c_2 = 0$

$y'(0) = 2c_1 - 2c_2 = 10$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & -2 & 10 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & -1 & 5 \end{array}\right]$$

$$-R_1+R_2 \quad \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -2 & 2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -1 \end{array}\right]$$

$c_1 = 4$

$c_2 = -1$

$$\boxed{y(x) = -3 + 4e^{2x} - e^{-2x}}$$

26) $y'' + 2y = 4$ $\quad y_p = 2$

$y'' + 2y = 6x$ $\quad y_p = 3x$

so $\boxed{y_p = 2 + 3x}$ solves $y'' + 2y = 4 + 6x$