Math 2250

HW due 9/26

3.3 13, 15, 18, 32

3.4 3, 5, 7, 10, 13, 19, 31, 32, 34, 39, 44

3.5 5, 7, 8, 13, 23, 30, 31, 32, 33

3.3 13)

$$\begin{array}{c|cccc} & 2 & 7 & 4 & 0 \\ & 1 & 3 & 2 & 1 \\ & 2 & 6 & 5 & 4 \\ \hline R_1 & 1 & 3 & 2 & 1 \\ R_2 & 2 & 7 & 4 & 0 \\ R_3 & 2 & 6 & 5 & 4 \\ \hline & 1 & 3 & 2 & 1 \\ -2R_1+R_2 & 0 & 1 & 0 & -2 \\ -2R_1+R_3 & 0 & 0 & 1 & 2 \\ \hline -2R_3+R_1 & 1 & 3 & 0 & -3 \\ & 0 & 1 & 0 & -2 \\ & 0 & 0 & 1 & 2 \\ \hline -3R_2+R_1 & 1 & 0 & 0 & 3 \\ & 0 & 1 & 0 & -2 \\ & 0 & 0 & 1 & 2 \end{array}$$

rref(A)

18)

$$\begin{array}{c|ccccc} & 1 & -2 & -5 & -12 & 1 \\ & 2 & 3 & 18 & 11 & 9 \\ & 2 & 5 & 26 & 21 & 11 \\ \hline & 1 & -2 & -5 & -12 & 1 \\ -2R_1+R_2 & 0 & 7 & 28 & 35 & 7 \\ -2R_1+R_3 & 0 & 9 & 36 & 45 & 9 \\ \hline & 1 & -2 & -5 & -12 & 1 \\ R_2/7 & 0 & 1 & 4 & 5 & 1 \\ R_3/9 & 0 & 1 & 4 & 5 & 1 \\ \hline & 1 & -2 & -5 & -12 & 1 \\ & 0 & 1 & 4 & 5 & 1 \\ -R_3+R_2 & 0 & 0 & 0 & 0 & 0 \\ \hline 2R_2+R_1 & 1 & 0 & 3 & -2 & 3 \\ & 0 & 1 & 4 & 5 & 1 \\ & 0 & 0 & 0 & 0 & 0 \end{array}$$

rref(A)

32) $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

if $a=0$, then $c \neq 0$ if $ad-bc \neq 0$

if $a \neq 0$

$$\begin{array}{c|cc} \frac{1}{a}R_1 & 1 & b/a \\ & c & d \\ \hline & 1 & b/a \\ -cR_1+R_2 & 0 & d-\frac{cb}{a} \\ \hline & 1 & b/a \\ & 0 & \frac{ad-bc}{a} \\ \hline & 1 & b/a \\ & 0 & 1 \end{array}$$

so if det $= ad-bc \neq 0$

this term is non-zero, so divide by it to get a 1

$$\begin{array}{c|cc} R_2 & c & d \\ R_1 & a & b \\ \hline \frac{1}{c}R_1 & 1 & d/c \\ & a & b \\ \hline & 1 & d/c \\ -aR_1+R_2 & 0 & b-\frac{ad}{c} \\ \hline & 1 & d/c \\ & 0 & \frac{bc-ad}{c} \\ \hline & 1 & d/c \\ & 0 & 1 \end{array}$$

$$\begin{array}{c|cc} -\frac{d}{c}R_2+R_1 & 1 & 0 \\ & 0 & 1 \end{array}$$

3.4 #3) $-2\begin{bmatrix}5 & 0\\ 0 & 7\\ 3 & -1\end{bmatrix} + 4\begin{bmatrix}-4 & 5\\ 3 & 2\\ 7 & 4\end{bmatrix} = \begin{bmatrix}-26 & 20\\ 12 & -6\\ 22 & 18\end{bmatrix}$

5) $AB = \begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}\begin{bmatrix}-4 & 2\\ 1 & 3\end{bmatrix} = \begin{bmatrix}-8-1 & 4-3\\ -12+2 & 6+6\end{bmatrix} = \begin{bmatrix}-9 & 1\\ -10 & 12\end{bmatrix}$; $BA = \begin{bmatrix}-4 & 2\\ 1 & 3\end{bmatrix}\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix} = \begin{bmatrix}-2 & 8\\ 11 & 5\end{bmatrix}$

7) $AB = [1\ \ 2\ \ 3]\begin{bmatrix}3\\ 4\\ 5\end{bmatrix} = [3+8+15] = [26]$; $BA = \begin{bmatrix}3\\ 4\\ 5\end{bmatrix}[1\ \ 2\ \ 3] = \begin{bmatrix}3 & 6 & 9\\ 4 & 8 & 12\\ 5 & 10 & 15\end{bmatrix}$

10) $AB = \begin{bmatrix}2 & 1\\ 4 & 3\end{bmatrix}\begin{bmatrix}-1 & 0 & 4\\ 3 & -2 & 5\end{bmatrix} = \begin{bmatrix}1 & -2 & 13\\ 5 & -6 & 31\end{bmatrix}$; $\cancel{BA = \begin{bmatrix}-1 & 0 & 4\\ 3 & -2 & 5\end{bmatrix}\begin{bmatrix}2 & 1\\ 4 & 3\end{bmatrix}}$ DNE [Does not exist].

13) $A(BC) = \begin{bmatrix}3 & 1\\ -1 & 4\end{bmatrix}\left(\begin{bmatrix}2 & 5\\ -3 & 1\end{bmatrix}\begin{bmatrix}0 & 1\\ 2 & 3\end{bmatrix}\right)$

$= \begin{bmatrix}3 & 1\\ -1 & 4\end{bmatrix}\begin{bmatrix}10 & 17\\ 2 & 0\end{bmatrix}$

$= \begin{bmatrix}32 & 51\\ -2 & -17\end{bmatrix}$

$(AB)C = \left(\begin{bmatrix}3 & 1\\ -1 & 4\end{bmatrix}\begin{bmatrix}2 & 5\\ -3 & 1\end{bmatrix}\right)\begin{bmatrix}0 & 1\\ 2 & 3\end{bmatrix}$

$= \begin{bmatrix}3 & 16\\ -14 & -1\end{bmatrix}\begin{bmatrix}0 & 1\\ 2 & 3\end{bmatrix}$

$= \begin{bmatrix}32 & 51\\ -2 & -17\end{bmatrix}$

EQUAL!

19) $\begin{bmatrix}1 & 0 & 0 & 3 & -1\\ 0 & 1 & 0 & -2 & 6\\ 0 & 0 & 1 & 1 & -8\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$

$x_5 = t$
$x_4 = s$
$x_3 = 8t - s$
$x_2 = -6t + 2s$
$x_1 = t - 3s$

$\vec{x} = t\begin{bmatrix}1\\ -6\\ 8\\ 0\\ 1\end{bmatrix} + s\begin{bmatrix}-3\\ 2\\ -1\\ 1\\ 0\end{bmatrix}$.

31) $(A+B)(A-B) = \left(\begin{bmatrix}2 & -1\\ -4 & 3\end{bmatrix} + \begin{bmatrix}1 & 5\\ 3 & 7\end{bmatrix}\right)(A-B) = \begin{bmatrix}3 & 4\\ -1 & 10\end{bmatrix}\begin{bmatrix}1 & -6\\ -7 & -4\end{bmatrix} = \begin{bmatrix}-25 & -34\\ -71 & -36\end{bmatrix}$ ← NOT EQUAL

$A^2 - B^2 = \begin{bmatrix}2 & -1\\ -4 & 3\end{bmatrix}\begin{bmatrix}2 & -1\\ -4 & 3\end{bmatrix} - \begin{bmatrix}1 & 5\\ 3 & 7\end{bmatrix}\begin{bmatrix}1 & 5\\ 3 & 7\end{bmatrix} = \begin{bmatrix}8 & -5\\ -20 & 13\end{bmatrix} - \begin{bmatrix}16 & 40\\ 24 & 64\end{bmatrix} = \begin{bmatrix}-8 & -45\\ -44 & -51\end{bmatrix}$

31b) $(A+B)(A-B)$
$= (A+B)A + (A+B)(-B)$ mult distributes over +
$= A^2 + BA - AB - B^2$ " (& scalars factor out. P. 179

So it is always true that
$(A+B)(A-B) = A^2 + BA - AB - B^2$

so if AB=BA the inside terms cancel, any you are left with

$A^2 - B^2$ ∎

§3.4 #34) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$

$= \begin{bmatrix} a^2+bc & b(a+d) \\ c(a+d) & bc+d^2 \end{bmatrix}$

if a,b,c,d are ±1, then $c(a+d) = b(a+d) = 0 \Rightarrow \boxed{a=-d}$ (off diagonal terms)

also $a^2+bc = 0 \Rightarrow 1+bc=0 \Rightarrow \boxed{b=-c}$ (since. $b,c=\pm 1$)

e.g $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$, $\begin{bmatrix} +1 & -1 \\ 1 & -1 \end{bmatrix}$, $\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$. (you only needed 1 such matrix)

1

39) if $A\vec{x}_1 = \vec{0}$

and $A\vec{x}_2 = \vec{0}$

then $A(c_1\vec{x}_1 + c_2\vec{x}_2) = A(c_1\vec{x}_1) + A(c_2\vec{x}_2)$ mult distributes over +

$= c_1 A\vec{x}_1 + c_2 A\vec{x}_2$ scalars factor out

$= c_1\vec{0} + c_2\vec{0} = \vec{0}$ ∎

§3.5 5) $A = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$ so $A^{-1} = \frac{1}{12-10}\begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix}$, so $\vec{x} = A^{-1}\vec{b}$

$= \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 8 \\ -7 \end{bmatrix} = \boxed{\begin{bmatrix} 4 \\ -7/2 \end{bmatrix}}$

check $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ -7/2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$ ✓

2

7) $A = \begin{bmatrix} 7 & 9 \\ 5 & 7 \end{bmatrix}$

so $A^{-1} = \frac{1}{49-45}\begin{bmatrix} 7 & -9 \\ -5 & 7 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & -9 \\ -5 & 7 \end{bmatrix}$

so $\vec{x} = A^{-1}\vec{b}$

$= \frac{1}{4}\begin{bmatrix} 7 & -9 \\ -5 & 7 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \boxed{\frac{1}{4}\begin{bmatrix} 3 \\ -1 \end{bmatrix}}$

check: $\begin{bmatrix} 7 & 9 \\ 5 & 7 \end{bmatrix}\frac{1}{4}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12 \\ 8 \end{bmatrix}$

$= \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ ✓

2

8) $A = \begin{bmatrix} 8 & 15 \\ 5 & 10 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$A^{-1} = \frac{1}{5}\begin{bmatrix} 10 & -15 \\ -5 & 8 \end{bmatrix}$

$\vec{x} = A^{-1}\vec{b}$

$= \frac{1}{5}\begin{bmatrix} 10 & -15 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \end{bmatrix} = \boxed{\frac{1}{5}\begin{bmatrix} 25 \\ -11 \end{bmatrix}}$

check $\begin{bmatrix} 8 & 15 \\ 5 & 10 \end{bmatrix}\frac{1}{5}\begin{bmatrix} 25 \\ -11 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 200-165 \\ 125-110 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ ✓

2

13.
$$\left[\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 3 & 7 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_2 \\ R_1 \\ \\ \end{array}\left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 2 & 7 & 3 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} \\ -2R_1+R_2 \\ -3R_1+R_3 \end{array}\left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & -2 & 0 \\ 0 & -2 & 3 & 0 & -3 & 1 \end{array}\right]$$

$$\begin{array}{l} \\ \\ 2R_2+R_3 \end{array}\left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 2 & -7 & 1 \end{array}\right]$$

$$\begin{array}{l} -2R_3+R_1 \\ R_3+R_2 \\ \end{array}\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -4 & 15 & -2 \\ 0 & 1 & 0 & 3 & -9 & 1 \\ 0 & 0 & 1 & 2 & -7 & 1 \end{array}\right]$$

$$\begin{array}{l} -3R_2+R_1 \\ \\ \end{array}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -13 & 42 & -5 \\ 0 & 1 & 0 & 3 & -9 & 1 \\ 0 & 0 & 1 & 2 & -7 & 1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -13 & 42 & -5 \\ 3 & -9 & 1 \\ 2 & -7 & 1 \end{bmatrix}}$$

§3.4 #23) $AX = B$

$\Rightarrow (A^{-1}(A)X) = A^{-1}B$

$\Rightarrow IX = A^{-1}B$

$\boxed{X = A^{-1}B}$

2

$A = \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}$; $A^{-1} = \frac{1}{1}\begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$

$A^{-1}B = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 & -5 \\ -1 & -2 & 5 \end{bmatrix}$

$\boxed{X = \begin{bmatrix} 7 & 18 & -35 \\ -9 & -23 & 45 \end{bmatrix}}$

check $\begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} 7 & 18 & -35 \\ -9 & -23 & 45 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -5 \\ -1 & -2 & 5 \end{bmatrix}$ ✓

30) $B = A^{-1}$ iff $AB = BA = I$

so, does $(ABC)(C^{-1}B^{-1}A^{-1}) = I$?

$(C^{-1}B^{-1}A^{-1})(ABC) = I$?

by associating:

$AB(CC^{-1})B^{-1}A^{-1} = ABIB^{-1}A^{-1} = ABB^{-1}A^{-1} = AA^{-1} = I$ ✓

1

also $C^{-1}B^{-1}(A^{-1}A)BC = C^{-1}(B^{-1}B)C = C^{-1}C = I$ ✓

32) if $AB = AC$ and A^{-1} exists, then

$\Rightarrow A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow (A^{-1}A)B = (A^{-1}A)C$

2

$\Rightarrow IB = IC$

$\Rightarrow B = C$ ■

Math 2250-4
HW due Oct 3, 2001
3.6 3, 13, 17, 21, 33

3. $\begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 5 & 0 \\ 3 & 6 & 9 & 8 \\ 4 & 0 & 10 & 7 \end{vmatrix} = 1 \begin{vmatrix} 0 & 5 & 0 \\ 6 & 9 & 8 \\ 0 & 10 & 7 \end{vmatrix}$ across row_1

$= -6 \begin{vmatrix} 5 & 0 \\ 10 & 7 \end{vmatrix}$ down col_1

$= -6 \cdot 5 \cdot 7 = -210$ (lower triangular)

13 $\begin{vmatrix} -4 & 4 & -1 \\ -1 & -2 & 2 \\ 1 & 4 & 3 \end{vmatrix} = - \begin{vmatrix} 1 & 4 & 3 \\ -1 & -2 & 2 \\ -4 & 4 & -1 \end{vmatrix} \begin{matrix} R_3 \\ \\ R_1 \end{matrix}$

$= - \begin{vmatrix} 1 & 4 & 3 \\ 0 & 2 & 5 \\ 0 & 20 & 11 \end{vmatrix} \begin{matrix} \\ R_1+R_2 \\ 4R_1+R_3 \end{matrix}$

$= - \begin{vmatrix} 1 & 4 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & -39 \end{vmatrix} \begin{matrix} \\ \\ -10R_2+R_3 \end{matrix}$

$= -2(-39) = 78$ (upper triangular)

17. $\begin{vmatrix} 2 & 3 & 3 & 1 \\ 0 & 4 & 3 & -3 \\ 2 & -1 & -1 & -3 \\ 0 & -4 & -3 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 3 & 1 \\ 0 & 4 & 3 & -3 \\ 0 & -4 & -4 & -4 \\ 0 & -4 & -3 & 2 \end{vmatrix} \begin{matrix} \\ \\ -R_1+R_3 \\ \\ \end{matrix} = \begin{vmatrix} 2 & 3 & 3 & 1 \\ 0 & 4 & 3 & -3 \\ 0 & 0 & -1 & -7 \\ 0 & 0 & 0 & -1 \end{vmatrix} \begin{matrix} \\ \\ R_2+R_3 \\ R_2+R_4 \end{matrix}$ (triangular) $= 2 \cdot 4(-1)(-1) = 8$

21. $3x + 4y = 2$
$5x + 7y = 1$

Cramer: $x = \dfrac{\begin{vmatrix} 2 & 4 \\ 1 & 7 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix}} = \dfrac{10}{1} = 10$

$y = \dfrac{\begin{vmatrix} 3 & 2 \\ 5 & 1 \end{vmatrix}}{1} = -7$

$30 - 28 = 2$ ✓
$50 - 49 = 1$ ✓

33. $A = \begin{bmatrix} -5 & -2 & 2 \\ 1 & 5 & -3 \\ 5 & -3 & 1 \end{bmatrix}$

$\text{cof}(A) = \begin{bmatrix} \begin{vmatrix} 5 & -3 \\ -3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & -3 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 5 \\ 5 & -3 \end{vmatrix} \\ -\begin{vmatrix} -2 & 2 \\ -3 & 1 \end{vmatrix} & \begin{vmatrix} -5 & 2 \\ 5 & 1 \end{vmatrix} & -\begin{vmatrix} -5 & -2 \\ 5 & -3 \end{vmatrix} \\ \begin{vmatrix} -2 & 2 \\ 5 & -3 \end{vmatrix} & -\begin{vmatrix} -5 & 2 \\ 1 & -3 \end{vmatrix} & \begin{vmatrix} -5 & -2 \\ 1 & 5 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -4 & -8 & -28 \\ -4 & -15 & -25 \\ -4 & -13 & -23 \end{bmatrix}$; $\text{adj}(A) = \text{cof}(A)^T$

$= \begin{bmatrix} -4 & -4 & -4 \\ -8 & -15 & -13 \\ -28 & -25 & -23 \end{bmatrix}$

$|A| = \begin{vmatrix} 0 & -5 & 3 \\ 1 & 5 & -3 \\ 5 & -3 & 1 \end{vmatrix} R_3+R_1 = -\begin{vmatrix} -5 & 3 \\ -3 & 1 \end{vmatrix} + 5 \cancel{\begin{vmatrix} -5 & 3 \\ 5 & -3 \end{vmatrix}} \to 0$ (col 1)

$= -4$

$A^{-1} = \dfrac{1}{|A|} \text{adj}(A) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 15/4 & 13/4 \\ 7 & 25/4 & 23/4 \end{bmatrix}$