8. $\det(A-\lambda I) = \det\begin{bmatrix} 1-\lambda & 1 & 0 & 1 \\ 0 & 1-\lambda & 1 & 1 \\ 0 & 0 & 2-\lambda & 1 \\ 0 & 0 & 0 & 2-\lambda \end{bmatrix} = (1-\lambda)^2(2-\lambda)^2$

$\lambda = 1$:
$$\left[\begin{array}{cccc|c} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$
$$\left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$v_4 = 0$
$v_3 = 0$
$v_2 = 0$
$v_1 = t$

$\vec{v} = t\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 1$ E-space $= \text{span}(\vec{e}_1)$

$\lambda = 2$:
$$\left[\begin{array}{cccc|c} -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$
$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$v_4 = 0$
$v_3 = t$
$v_2 = t$
$v_1 = t$

$\vec{v} = t\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = 2$ E-space $= \text{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}\right\}$

only 2 lin. ind. eigenvectors – need 4!
A is NOT diagonalizable

7.1 3. $t^2x'' + tx' + (t^2-1)x = 0$ $\longleftrightarrow$ $x_1 = x$, $x_2 = x'$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{1}{t}x_2 + \frac{(1-t^2)}{t^2}x_1 \end{bmatrix}$$

$x_2' = x'' = -\frac{1}{t}x' + \frac{(1-t^2)}{t^2}x$

or $\begin{bmatrix} x_1' \\ t^2 x_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ -tx_2 + (1-t^2)x_1 \end{bmatrix}$

8. $x'' + 3x' + 4x - 2y = 0$
$y'' + 2y' - 3x + y = \cos t$

let $x_1 = x$
$x_2 = x'$
$y_1 = y$
$y_2 = y'$

$$\begin{bmatrix} x_1' \\ x_2' \\ y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ -4x_1 - 3x_2 + 2y_1 \\ y_2 \\ 3x_1 - y_1 - 2y_2 + \cos t \end{bmatrix}$$

$x_2' = x'' = -3x' - 4x + 2y$
$y_2' = y'' = -2y' + 3x - y + \cos t$

18. $x' = -y$
$y' = 10x - 7y$

$\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \end{bmatrix}$

this one can be converted into a 2nd order eqn:
$x' = -y$
so $x'' = -y' = -10x + 7y$
$= -10x - 7x'$

$x'' + 7x' + 10x = 0$

$x = e^{rt}$: $r^2 + 7r + 10 = 0$
$(r+5)(r+2) = 0$; $r = -2, -5$

$x = c_1 e^{-5t} + c_2 e^{-2t}$
$y = -x' = 5c_1 e^{-5t} + 2c_2 e^{-2t}$

$x(0) = 2 = c_1 + c_2$
$y(0) = -7 = 5c_1 + 2c_2$

$$c_1 = \frac{\begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix}} = \frac{11}{-3} = -\frac{11}{3}$$

$$c_2 = \frac{\begin{vmatrix} 1 & 2 \\ 5 & -7 \end{vmatrix}}{-3} = \frac{-17}{-3} = \frac{17}{3}$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = -\frac{11}{3}e^{-5t}\begin{bmatrix} 1 \\ 5 \end{bmatrix} + \frac{17}{3}e^{-2t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Note $\left\{\begin{array}{l} \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 10 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \\ \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \end{bmatrix} \end{array}\right.$

Could you do this using evals/evecs

§7.1 21. a)

11: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} y \\ -x \end{bmatrix}$ is one we did in class:

$x'' = y' = -x \qquad x'' + x = 0$

$x = C\cos(t-\alpha)$

$y = x' = -C\sin(t-\alpha)$

$\begin{bmatrix} x \\ y \end{bmatrix} = C\begin{bmatrix} \cos(t-\alpha) \\ -\sin(t-\alpha) \end{bmatrix}$ so $x^2+y^2 = C^2(\cos^2+\sin^2) = C^2$

so $\begin{bmatrix} x \\ y \end{bmatrix}$ lies on circle of radius C

shortcut:

$\frac{d}{dt}(x^2+y^2)$

$= 2xx' + 2yy'$

$= 2x(y) + 2y(-x)$ (from sys)

$= 0!$ so x^2+y^2 is const.

21b). 12. $x' = y$

$y' = x$

$x'' = y' = x$

$x'' - x = 0$

$x = e^{rt} \quad r^2 - 1 = 0; \quad r = \pm 1$

$x = c_1e^t + c_2e^{-t}$

$y = x' = c_1e^t - c_2e^{-t}$

$x^2 - y^2 = [c_1e^t + c_2e^{-t}]^2 - [c_1e^t - c_2e^{-t}]^2$

$\left.\begin{aligned} &= e^{2t}(c_1^2 - c_1^2) \\ &+ e^{-2t}(c_2^2 - c_2^2) \\ &+ 1\,(2c_1c_2 + 2c_1c_2) \end{aligned}\right\} = 4c_1c_2 = \text{const}!$

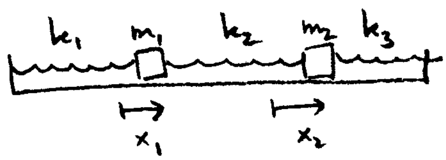
shortcut: $\frac{d}{dt}(x^2-y^2) = 2xx' - 2yy'$

$= 2x(y) - 2y(x)$, from D.E.

$= 0$

so $x^2 - y^2 \equiv$ const.

24.

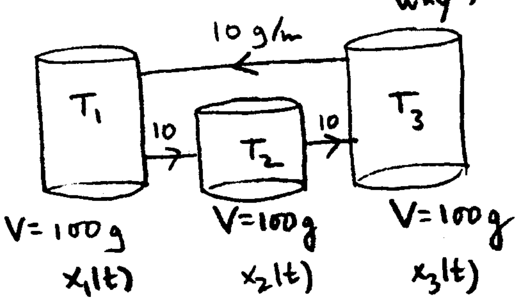


$m_1x_1'' = k_2(x_2-x_1) - k_1x_1 \quad = x_1(-k_1-k_2) + x_2k_2$

$m_2x_2'' = -k_2(x_2-x_1) - k_3x_2 \quad = x_1k_2 + x_2(-k_2-k_3)$ ✓

why?

26.



each rate is 10 g/m

r_i · tank 3 concentration · r_o · tank 1 concent

$\frac{dx_1}{dt} = 10\left(\frac{x_3}{100}\right) - 10\left(\frac{x_1}{100}\right)$

Similarly: $\frac{dx_2}{dt} = 10\frac{x_1}{100} - 10\frac{x_2}{100}$

$\frac{dx_3}{dt} = 10\frac{x_2}{100} - 10\frac{x_3}{100}$

if you multiply each eqtn by 10 you get the system in the book.

Math 2250-4
HW due Weds 11/21

7.2 5, 9, 10, 13, 14, 16, 21, 23, 25
7.3 3, 4, 6, 13, 14, 25, 30, 32

9. $x' = 3x - 4y + z + t$
$y' = x - 3z + t^2$
$z' = 6y - 7z + t^3$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}' = \begin{bmatrix} 3 & -4 & 1 \\ 1 & 0 & -3 \\ 0 & 6 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} t \\ t^2 \\ t^3 \end{bmatrix}$$

10. $x' = tx - y + e^t z$
$y' = 2x + t^2 y - z$
$z' = e^{-t} x + 3ty + t^3 z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}' = \begin{bmatrix} t & -1 & e^t \\ 2 & t^2 & -1 \\ e^{-t} & 3t & t^3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

14. $\vec{x}' = \begin{bmatrix} -3 & 2 \\ -3 & 4 \end{bmatrix} \vec{x}$ $\quad \vec{x}_1 = \begin{bmatrix} e^{3t} \\ 3e^{3t} \end{bmatrix}$ $\quad \vec{x}_1' = \begin{bmatrix} 3e^{3t} \\ 9e^{3t} \end{bmatrix}$

$$A\vec{x}_1 = \begin{bmatrix} -3 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} e^{3t} \\ 3e^{3t} \end{bmatrix} = \begin{bmatrix} (-3+6)e^{3t} \\ (-3+12)e^{3t} \end{bmatrix} = \begin{bmatrix} 3e^{3t} \\ 9e^{3t} \end{bmatrix} \checkmark$$

$\vec{x}_2 = \begin{bmatrix} 2e^{-2t} \\ e^{-2t} \end{bmatrix}$ $\quad \vec{x}_2' = \begin{bmatrix} -4e^{-2t} \\ -2e^{-2t} \end{bmatrix}$

$$A\vec{x}_2 = \begin{bmatrix} -3 & 2 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 2e^{-2t} \\ e^{-2t} \end{bmatrix} = \begin{bmatrix} e^{-2t}(-6+2) \\ e^{-2t}(-6+4) \end{bmatrix} = \begin{bmatrix} -4e^{-2t} \\ -2e^{-2t} \end{bmatrix} \checkmark$$

$$W = \left| \vec{x}_1 \; \vec{x}_2 \right| = \begin{vmatrix} e^{3t} & 2e^{-2t} \\ 3e^{3t} & e^{-2t} \end{vmatrix}$$

$$= e^{3t} e^{-2t} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = \boxed{e^t(-5)} \neq 0$$ so soltns are lin ind.

↑ from 1st col ↑ from 2nd col

so $$\boxed{\vec{x}_H(t) = c_1 e^{3t} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}}$$

16. $\vec{x}' = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} \vec{x}$

$\vec{x}_1 = e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\quad \vec{x}_2 = e^{2t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$; $\vec{x}_2' = 2e^{2t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\vec{x}_1' = 3e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$A\vec{x}_2 = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} e^{2t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = e^{2t} \begin{bmatrix} 2 \\ -4 \end{bmatrix} = 2e^{2t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$A\vec{x}_1 = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = e^{3t} \begin{bmatrix} 3 \\ -3 \end{bmatrix} = 3e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$W = \det \left[e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}, e^{2t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right] = e^{3t} e^{2t} \begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} = \boxed{e^{5t}(-1)} \neq 0$$ so l.i.

$$\boxed{\vec{x}_H(t) = c_1 e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}}$$

$$\vec{x}\,' = \begin{bmatrix} -8 & -11 & -2 \\ 6 & 9 & 2 \\ -6 & -6 & 1 \end{bmatrix} \vec{x}$$

$$\vec{x}_1 = e^{-2t}\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$$

$$\vec{x}_1\,' = -2e^{-2t}\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix} \qquad A\vec{x}_1 = \begin{bmatrix} -8 & -11 & -2 \\ 6 & 9 & 2 \\ -6 & -6 & 1 \end{bmatrix} e^{-2t}\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$$

$$= e^{-2t}\begin{bmatrix} -24+22-4 \\ 18-18+4 \\ -18+12+2 \end{bmatrix} = e^{-2t}\begin{bmatrix} -6 \\ 4 \\ -4 \end{bmatrix}$$

$$\vec{x}_2 = e^{t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{x}_2\,' = e^{t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \qquad A\vec{x}_2 = \begin{bmatrix} -8 & -11 & -2 \\ 6 & 9 & 2 \\ -6 & -6 & 1 \end{bmatrix} e^{t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$= e^{t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{x}_3 = e^{3t}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\vec{x}_3\,' = 3e^{3t}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$A\vec{x}_3 = \begin{bmatrix} -8 & -11 & -2 \\ 6 & 9 & 2 \\ -6 & -6 & 1 \end{bmatrix} e^{3t}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = e^{3t}\begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$$

$$W = \left| e^{-2t}\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix} \;\; e^{t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \;\; e^{3t}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right|$$

$$= e^{-2t}e^{t}e^{3t}\begin{vmatrix} 3 & 1 & 1 \\ -2 & -1 & -1 \\ 2 & 1 & 0 \end{vmatrix} = e^{2t}(-2-2+2+3)$$

$$= e^{2t} \neq 0$$

So $\boxed{\vec{x}_H(t) = c_1 e^{-2t}\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix} + c_2 e^{t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + c_3 e^{3t}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}$

23. refer to #14:

$$c_1\begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} -10 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

So $\vec{x}_H(t) = 2e^{3t}\begin{bmatrix} 1 \\ 3 \end{bmatrix} - e^{-2t}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

25. refer to 16

$$\begin{bmatrix} 11 \\ -7 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 11 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 11 \\ -7 \end{bmatrix} = \begin{bmatrix} 15 \\ -4 \end{bmatrix}$$

So $\vec{x}_H(t) = 15e^{3t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} - 4e^{2t}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

3 #4) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 4 & 1 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$|A - \lambda I| = \begin{vmatrix} 4-\lambda & 1 \\ 6 & -1-\lambda \end{vmatrix} = \lambda^2 - 3\lambda - 10 = (\lambda - 5)(\lambda + 2)$$

$\lambda = 5, -2.$

$\lambda = 5$:

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 6 & -6 & 0 \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

$\vec{z}_1 = e^{5t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

$\lambda = -2$

$$\left[\begin{array}{cc|c} 6 & 1 & 0 \\ 6 & 1 & 0 \end{array}\right] \quad \left[\begin{array}{cc|c} 6 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\vec{v} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

$\vec{z}_2 = e^{-2t} \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

$$\boxed{\vec{x}_H(t) = c_1 e^{5t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} -1 \\ 6 \end{bmatrix}}$$

6) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 9 & 5 \\ -6 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$|A - \lambda I| = \begin{vmatrix} 9-\lambda & 5 \\ -6 & -2-\lambda \end{vmatrix} = \lambda^2 - 7\lambda + 12 = (\lambda - 4)(\lambda - 3)$$

$\lambda = 3, 4$

$\lambda = 3$

$$\left[\begin{array}{cc|c} 6 & 5 & 0 \\ -6 & -5 & 0 \end{array}\right] \quad \left[\begin{array}{cc|c} 6 & 5 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\vec{v} = \begin{bmatrix} -5 \\ 6 \end{bmatrix}.$

$\lambda = 4$

$$\left[\begin{array}{cc|c} 5 & 5 & 0 \\ -6 & -6 & 0 \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$

$$\vec{x}_H(t) = c_1 e^{3t} \begin{bmatrix} -5 \\ 6 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\vec{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 1 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \end{bmatrix}$$

$$\boxed{\vec{x}(t) = e^{3t} \begin{bmatrix} -5 \\ 6 \end{bmatrix} - 6e^{4t} \begin{bmatrix} -1 \\ 1 \end{bmatrix}}$$

14) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 + 16 = (3 - \lambda - 4i)(3 - \lambda + 4i) = 0$$

$\lambda = 3 \pm 4i$

$\lambda = 3 + 4i$

$$\left[\begin{array}{cc|c} -4i & -4 & 0 \\ 4 & -4i & 0 \end{array}\right] \quad \left[\begin{array}{cc|c} i & 1 & 0 \\ 1 & -i & 0 \end{array}\right]$$

$$\begin{array}{c} R_2 \\ R_1 \end{array} \left[\begin{array}{cc|c} 1 & -i & 0 \\ i & 1 & 0 \end{array}\right] \quad \begin{array}{c} \\ -iR_1 + R_2 \end{array} \left[\begin{array}{cc|c} 1 & -i & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\vec{v} = \begin{bmatrix} i \\ 1 \end{bmatrix}.$

$$e^{\lambda t} \vec{v} = e^{(3+4i)t} \begin{bmatrix} i \\ 1 \end{bmatrix}$$

$$= e^{3t} (\cos 4t + i \sin 4t) \begin{bmatrix} i \\ 1 \end{bmatrix}$$

real part is soltn

Im part is another one!

$$= e^{3t} \begin{bmatrix} -\sin 4t \\ \cos 4t \end{bmatrix} + i\, e^{3t} \begin{bmatrix} \cos 4t \\ \sin 4t \end{bmatrix}$$

$$\boxed{\vec{x}_H(t) = e^{3t} \left[c_1 \begin{bmatrix} -\sin 4t \\ \cos 4t \end{bmatrix} + c_2 \begin{bmatrix} \cos 4t \\ \sin 4t \end{bmatrix} \right]}$$

3 #30 $\frac{dx_1}{dt} = -10\frac{x_1}{25} + 10\frac{x_2}{40}$

$\frac{dx_2}{dt} = 10\frac{x_1}{25} - 10\frac{x_2}{40}$

(5)

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -.4 & .25 \\ .4 & -.25 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

$$|A-\lambda I| = \begin{vmatrix} -.4-\lambda & .25 \\ .4 & -.25-\lambda \end{vmatrix} = \lambda^2 + .65\lambda + .4(.25) - .4(.25)$$

$$= \lambda(\lambda + .65)$$

$\lambda = 0$:

$$\left[\begin{array}{cc|c} -.4 & .25 & 0 \\ .4 & -.25 & 0 \\ \hline -40 & 25 & 0 \\ 0 & 0 & 0 \\ \hline -8 & 5 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\vec{v} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

$\lambda = -.65$

$$\left[\begin{array}{cc|c} .25 & .25 & 0 \\ .4 & .4 & 0 \\ \hline 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

limiting values make sense!

$$\boxed{\vec{x}(t) = \frac{15}{13}\begin{bmatrix} 5 \\ 8 \end{bmatrix} + \frac{120}{13}e^{-.65t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$\vec{x}(t) = c_1\begin{bmatrix} 5 \\ 8 \end{bmatrix} + c_2 e^{-.65t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\vec{x}(0) = \begin{bmatrix} 15 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 8 & -1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix};$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = -\frac{1}{13}\begin{bmatrix} -1 & -1 \\ -8 & 5 \end{bmatrix}\begin{bmatrix} 15 \\ 0 \end{bmatrix} = \begin{bmatrix} 15/13 \\ 120/13 \end{bmatrix}$$

#32 $r = 60$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 45 \\ 0 \\ 0 \end{bmatrix}$$

$V_1 = 20$, $V_2 = 30$, $V_3 = 60$

$\frac{dx_1}{dt} = -60\frac{x_1}{20}$

$\frac{dx_2}{dt} = 60\frac{x_1}{20} - 60\frac{x_2}{30}$

$\frac{dx_3}{dt} = 60\frac{x_2}{30} - 60\frac{x_3}{60}$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -2 & 0 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

matrix is triangular so evals are $-3, -2, -1$

$\lambda = -1$

$\vec{v} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda = -2$

$A - \lambda I$:

$$\left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$v_3 = t$
$v_2 = -\frac{1}{2}t$
$v_1 = 0$

$\vec{v} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$ ✓

$\lambda = -3$:

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ \hline 3 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$v_3 = t$
$v_2 = -t$
$v_1 = \frac{1}{3}t$

$\vec{v} = t\begin{bmatrix} 1/3 \\ -1 \\ 1 \end{bmatrix}$

$\vec{v} = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$ ✓

so $\vec{x}(t) = c_1 e^{-t}\begin{bmatrix}0\\0\\1\end{bmatrix} + c_2 e^{-2t}\begin{bmatrix}0\\-1\\2\end{bmatrix} + c_3 e^{-3t}\begin{bmatrix}1\\-3\\3\end{bmatrix}$

$\vec{x}(0) = \begin{bmatrix}45\\0\\0\end{bmatrix} = c_1\begin{bmatrix}0\\0\\1\end{bmatrix} + c_2\begin{bmatrix}0\\-1\\2\end{bmatrix} + c_3\begin{bmatrix}1\\-3\\3\end{bmatrix}$ $\Rightarrow$ $c_3 = 45$

$-c_2 - 135 = 0 \quad c_2 = -135$

$c_1 - 270 + 35 = 0 \quad c_1 = 135$

$\vec{x}(t) = 135 e^{-t}\begin{bmatrix}0\\0\\1\end{bmatrix} - 135 e^{-2t}\begin{bmatrix}0\\-1\\2\end{bmatrix} + 45 e^{-3t}\begin{bmatrix}1\\-3\\3\end{bmatrix}$

in tank 3, $x_3(t) = 135 e^{-t} - 270 e^{-2t} + 135 e^{-3t}$

Max when $\left(\frac{x_3}{135}\right)'(t) = 0$:

$$-e^{-t} + 4e^{-2t} - 3e^{-3t} = 0$$
$$-1 + 4e^{-t} - 3e^{-2t} = 0$$
$$3e^{-2t} - 4e^{-t} + 1 = 0$$
$$3z^2 - 4z + 1 = 0$$
$$(3z-1)(z-1) = 0$$
$$\uparrow_{\,t=0}$$

$z = \frac{1}{3}$; $e^{-t} = 1/3$

so $x_3(t) = \frac{135}{3} - \frac{270}{9} + \frac{\cancel{135}}{2\cancel{7}}\;\frac{45}{9}\;5$

$= 45 - 30 + 5$

$= \underline{\underline{20}}$ lbs.

it would be fine to use Maple! "solve"

```
:=t->135*(exp(-t)-2*exp(-2*t)+exp(-3*t));
```

$x3 := t \to 135\,e^{(-t)} - 270\,e^{(-2t)} + 135\,e^{(-3t)}$

```
olve(diff(x3(t),t)=0,t);
```

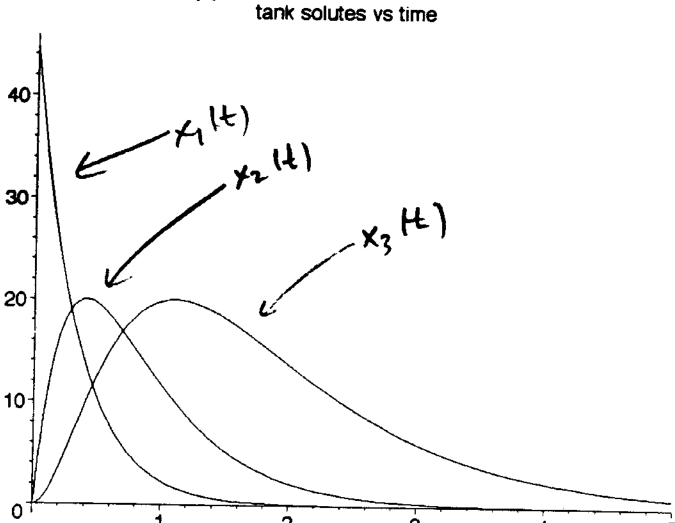
ln(3), 0

```
3(ln(3));
```

20

```
> plot({x1(t),x2(t),x3(t)},t=0..5,color=black,title=
  `tank solutes vs time`);
```

tank solutes vs time




```
> A:=matrix(3,3,[-3,0,0,3,-2,0,0,2,-1]);
```

$A := \begin{bmatrix}-3&0&0\\3&-2&0\\0&2&-1\end{bmatrix}$

```
> eigenvects(A);
```

[-3, 1, {[1, -3, 3]}], [-1, 1, {[0, 0, 1]}], [-2, 1, {[0, 1, -2]}]

So $\vec{x}(t) = c_1 e^{-t}\begin{bmatrix}0\\0\\1\end{bmatrix} + c_2 e^{-2t}\begin{bmatrix}0\\-1\\2\end{bmatrix} + c_3 e^{-3t}\begin{bmatrix}1\\-3\\3\end{bmatrix}$

$\vec{x}(0) = \begin{bmatrix}45\\0\\0\end{bmatrix} = c_1\begin{bmatrix}0\\0\\1\end{bmatrix} + c_2\begin{bmatrix}0\\-1\\2\end{bmatrix} + c_3\begin{bmatrix}1\\-3\\3\end{bmatrix}$ $\Rightarrow$ $c_3 = 45$

$-c_2 - 135 = 0 \quad c_2 = -135$

$c_1 - 270 + 135 = 0 \quad c_1 = 135$

$\vec{x}(t) = 135e^{-t}\begin{bmatrix}0\\0\\1\end{bmatrix} - 135e^{-2t}\begin{bmatrix}0\\-1\\2\end{bmatrix} + 45e^{-3t}\begin{bmatrix}1\\-3\\3\end{bmatrix}$

in tank 3, $x_3(t) = 135e^{-t} - 270e^{-2t} + 135e^{-3t}$

Max when $\left(\frac{x_3}{135}\right)'(t) = 0$:

$-e^{-t} + 4e^{-2t} - 3e^{-3t} = 0$

$-1 + 4e^{-t} - 3e^{-2t} = 0$

$3e^{-2t} - 4e^{-t} + 1 = 0$

$3z^2 - 4z + 1 = 0$

$(3z-1)(z-1) = 0$

$\uparrow$ $t=0$

$z = \frac{1}{3}$; $e^{-t} = 1/3$

so $x_3(t) = \frac{135}{3} - \frac{270}{9} + \frac{\cancel{135}}{\cancel{27}}\ \frac{45}{9}\ 5$

$= 45 - 30 + 5$

$= \underline{\underline{20}}$ lbs.

it would be fine to use Maple! "solve"

```
:=t->135*(exp(-t)-2*exp(-2*t)+exp(-3*t));
```

$x3 := t \to 135\,e^{(-t)} - 270\,e^{(-2t)} + 135\,e^{(-3t)}$

```
olve(diff(x3(t),t)=0,t);
```

ln(3), 0

```
(ln(3));
```

20

```
> A:=matrix(3,3,[-3,0,0,3,-2,0,0,2,-1]);
```

$A := \begin{bmatrix}-3 & 0 & 0\\3 & -2 & 0\\0 & 2 & -1\end{bmatrix}$

```
> eigenvects(A);
```

[-3, 1, {[1, -3, 3]}], [-1, 1, {[0, 0, 1]}], [-2, 1, {[0, 1, -2]}]

```
> plot({x1(t),x2(t),x3(t)},t=0..5,color=black,title=
  `tank solutes vs time`);
```

tank solutes vs time