Math 2250-4

Homework due Wed 11/14

6.1 (7, 17, 18, 24, 29)

6.2 3, 5, (6, 10) 15 (17) 27 (28)

7.1 1, (3) 5, (8) 11, (18) (21) (24) (26)

7. $|A-\lambda I| = \det\begin{bmatrix} 10-\lambda & -8 \\ 6 & -4-\lambda \end{bmatrix} = (\lambda+4)(\lambda-10)+48$

$= \lambda^2 - 6\lambda + 8$

$= (\lambda-4)(\lambda-2)$

evals $\lambda = 2, 4$

$\lambda = 2$:

$$\left[\begin{array}{cc|c} 8 & -8 & 0 \\ 6 & -6 & 0 \\ \hline 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] *$$

(read off $\vec{v} = \begin{bmatrix}1\\1\end{bmatrix}$, since we want $[1, -1]\begin{bmatrix}v_1\\v_2\end{bmatrix} = 0$

we can use "flip & minus" trick)

else, backsolve *

$v_2 = t$

$v_1 = t$ $\quad \vec{v} = t\begin{bmatrix}1\\1\end{bmatrix}$

basis $\begin{bmatrix}1\\1\end{bmatrix}$.

Check: $\begin{bmatrix}10 & -8\\6 & -4\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}$ ✓

$\lambda = 4$

$$\left[\begin{array}{cc|c} 6 & -8 & 0 \\ 6 & -8 & 0 \\ \hline 3 & -4 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\vec{v} = \begin{bmatrix}4\\3\end{bmatrix}$

check $\begin{bmatrix}10 & -8\\6 & -4\end{bmatrix}\begin{bmatrix}4\\3\end{bmatrix} = \begin{bmatrix}16\\12\end{bmatrix} = 4\begin{bmatrix}4\\3\end{bmatrix}$ ✓

ans: $\lambda = 2,\ \vec{v} = \begin{bmatrix}1\\1\end{bmatrix}$

$\lambda = 4,\ \vec{v} = \begin{bmatrix}4\\3\end{bmatrix}$

17. $|A-\lambda I| = \det\begin{bmatrix} 3-\lambda & 5 & -2 \\ 0 & 2-\lambda & 0 \\ 0 & 2 & 1-\lambda \end{bmatrix}$

$= (3-\lambda)(2-\lambda)(1-\lambda) + 0$

$\lambda = 3$:

$$\left[\begin{array}{ccc|c} 0 & 5 & -2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$v_3 = 0$

$v_2 = 0$ $\quad \vec{v} = t\begin{bmatrix}1\\0\\0\end{bmatrix}$.

$v_1 = t$

eigenpair $\left(3, \begin{bmatrix}1\\0\\0\end{bmatrix}\right)$

$\lambda = 2$:

$$\left[\begin{array}{ccc|c} 1 & 5 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ \hline 1 & 5 & -2 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ \hline 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\frac{1}{2}R_3$

R_2

$-5R_2 + R_1$

$v_3 = t$

$v_2 = \frac{1}{2}t$ $\quad \vec{v} = t\begin{bmatrix}-\frac{1}{2}\\ \frac{1}{2}\\ 1\end{bmatrix}$

$v_1 = -\frac{1}{2}t$

eigenpair $\left(2, \begin{bmatrix}-1\\1\\2\end{bmatrix}\right)$

$\lambda = 1$:

$$\left[\begin{array}{ccc|c} 2 & 5 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ \hline 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$v_3 = t$

$v_2 = 0$ $\quad \vec{v} = t\begin{bmatrix}1\\0\\1\end{bmatrix}$

$v_1 = t$

eigenpair $\left(1, \begin{bmatrix}1\\0\\1\end{bmatrix}\right)$

18 $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ -6 & 8-\lambda & 2 \\ 12 & -15 & -3-\lambda \end{vmatrix} = (1-\lambda)(8-\lambda)(-3-\lambda) + 30(1-\lambda)$

$= (1-\lambda)\left[(8-\lambda)(-3-\lambda)+30\right]$

$= (1-\lambda)\left[\lambda^2-5\lambda+6\right]$

$= (1-\lambda)(\lambda-2)(\lambda-3)$;

$\lambda = 1, 2, 3.$

$\lambda=1$: $\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -6 & 7 & 2 & 0 \\ 12 & -15 & -4 & 0 \end{array}\right]$

$2R_2+R_3$ $\left[\begin{array}{ccc|c} 12 & -15 & -4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 12 & 0 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$v_3 = t$
$v_2 = 0$
$v_1 = \frac{1}{3}t$

$\vec{v} = t\begin{bmatrix} 1/3 \\ 0 \\ 1 \end{bmatrix}$

eigenpair $\left(1, \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}\right)$

$\lambda=2$: $\left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ -6 & 6 & 2 & 0 \\ 12 & -15 & -5 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 6 & 2 & 0 \\ 0 & -15 & -5 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$v_3 = t$
$v_2 = -\frac{1}{3}t$
$v_1 = 0$

$\vec{v} = t\begin{bmatrix} 0 \\ -1/3 \\ 1 \end{bmatrix}$

eigenpair $\left(2, \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}\right)$

$\lambda=3$: $\left[\begin{array}{ccc|c} -2 & 0 & 0 & 0 \\ -6 & 5 & 2 & 0 \\ 12 & -15 & -6 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 5 & 2 & 0 \\ 0 & -15 & -6 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2/5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$v_3 = t$
$v_2 = -\frac{2}{5}t$
$v_1 = 0$

$\vec{v} = t\begin{bmatrix} 0 \\ -2/5 \\ 1 \end{bmatrix}$

eigenpair $\left(3, \begin{bmatrix} 0 \\ -2 \\ 5 \end{bmatrix}\right)$

24. $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 4 & 0 \\ 0 & 1-\lambda & 4 & 0 \\ 0 & 0 & 3-\lambda & 0 \\ 0 & 0 & 0 & 3-\lambda \end{vmatrix} = (1-\lambda)^2(3-\lambda)^2$

$\lambda = 1, 3$

$\lambda=1$: $\left[\begin{array}{cccc|c} 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{array}\right]$

$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$v_4 = 0$
$v_3 = 0$
$v_2 = t$
$v_1 = s$

$\vec{v} = s\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda=1$ eigenbasis $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\right\}$

$\lambda=3$: $\left[\begin{array}{cccc|c} -2 & 0 & 4 & 0 & 0 \\ 0 & -2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$v_4 = t$
$v_3 = s$
$v_2 = 2s$
$v_1 = 2s$

$\vec{v} = s\begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda=3$ eigenbasis $\left\{\begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}$

29. $|A-\lambda I| = \begin{vmatrix} -\lambda & -3 \\ 12 & -\lambda \end{vmatrix} = \lambda^2+36 = (\lambda+6i)(\lambda-6i)$

$\lambda=6i$:

$\left[\begin{array}{cc|c} -6i & -3 & 0 \\ 12 & -6i & 0 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & -i/2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $R_2/12$, $\frac{1}{2}R_2+R_1$

$\vec{v} = \begin{bmatrix} i/2 \\ 1 \end{bmatrix}$, or $\begin{bmatrix} i \\ 2 \end{bmatrix}$

so for $\lambda=-6i$ will get $\vec{v} = \begin{bmatrix} -i \\ 2 \end{bmatrix}$.

eigenpair $\left(6i, \begin{bmatrix} i \\ 2 \end{bmatrix}\right)$, eigenpair $\left(-6i, \begin{bmatrix} -i \\ 2 \end{bmatrix}\right)$.

(5)

6.2 #6 $\det(A-\lambda I) = \det\begin{bmatrix} 10-\lambda & -6 \\ 12 & -7-\lambda \end{bmatrix} = \lambda^2 - 3\lambda + 2 = (\lambda-2)(\lambda-1)$

$\lambda=1:$ $\left[\begin{array}{cc|c} 9 & -6 & 0 \\ 12 & -8 & 0 \\ \hline 3 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\lambda=2:$ $\left[\begin{array}{cc|c} 8 & -6 & 0 \\ 12 & -9 & 0 \\ \hline 4 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\vec{u} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$P = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ then $P^{-1}AP = D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

check: $\frac{1}{-1}\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}\underbrace{\begin{bmatrix} 10 & -6 \\ 12 & -7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}}$

$- \underbrace{\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 3 & 8 \end{bmatrix}}$

$- \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ ✓

10). $\det(A-\lambda I) = \det\begin{bmatrix} 3-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix} = (3-\lambda)(1-\lambda)+1 = \lambda^2 - 4\lambda + 4 = (\lambda-2)^2$

eval $\lambda=2$

$\lambda=2:$

$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & -1 & 0 \\ \hline 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$v_2 = t$, $v_1 = t$ $\quad \vec{v} = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ eigenspace is only 1-d. No more evals!

NOT diagonalizable.

17) $\det(A-\lambda I) = \det\begin{bmatrix} 7-\lambda & -8 & 3 \\ 6 & -7-\lambda & 3 \\ 2 & -2 & 2-\lambda \end{bmatrix} = (7-\lambda)(-7-\lambda)(2-\lambda) - 48 - 36 + 6(7+\lambda) + 6(7-\lambda) + 48(2-\lambda)$

$= -\lambda^3 + \lambda^2(2-7+7) + \lambda(49-14+14+6-6-48)$

$+ 1(-98-48-36+42+42+96)$

$= -\lambda^3 + 2\lambda^2 + \lambda - 2$

$= (\lambda-2)(-\lambda^2+1) = (\lambda-2)(\lambda-1)(\lambda+1) = 0 \quad \lambda = 1, -1, 2$

$\lambda=1:$ $\left[\begin{array}{ccc|c} 6 & -8 & 3 & 0 \\ 6 & -8 & 3 & 0 \\ 2 & -2 & 1 & 0 \\ \hline 2 & -2 & 1 & 0 \\ 6 & -8 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 2 & -2 & 1 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ ($-3R_1+R_2$)

$v_3 = t$
$v_2 = 0$
$v_1 = -\frac{1}{2}t$
$\vec{v} = t\begin{bmatrix} -\frac{1}{2} \\ 0 \\ 1 \end{bmatrix}$

$\left(1, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}\right)$

$\lambda=-1:$ $\left[\begin{array}{ccc|c} 8 & -8 & 3 & 0 \\ 6 & -6 & 3 & 0 \\ 2 & -2 & 3 & 0 \\ \hline 1 & -1 & \frac{1}{2} & 0 \\ 8 & -8 & 3 & 0 \\ 2 & -2 & 3 & 0 \\ \hline 1 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 0 \\ \hline 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ ($\frac{1}{6}R_2$, R_1; $-8R_1+R_2$, $-2R_1+R_3$)

$v_3 = 0$
$v_2 = t$
$v_1 = t$
$\vec{v} = t\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\left(-1, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right)$

cont'd

$\lambda=2$:
$$\left[\begin{array}{ccc|c} 5 & -8 & 3 & 0 \\ 6 & -9 & 3 & 0 \\ 2 & -2 & 0 & 0 \end{array}\right]$$

$$\begin{array}{l} R3/2 \\ \\ \frac{1}{3}R_2 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 5 & -8 & 3 & 0 \\ 2 & -3 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ -5R_1+R_2 \\ -2R_1+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_2+R_1 \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rightarrow$ $v_3 = t$, $v_2 = t$, $v_1 = t$ $\quad \vec{v} = t\begin{bmatrix}1\\1\\1\end{bmatrix}$

$\left(2, \begin{bmatrix}1\\1\\1\end{bmatrix}\right)$

Summary: $\lambda=1 \quad \vec{u} = \begin{bmatrix}-1\\0\\2\end{bmatrix}$

$\lambda=-1 \quad \vec{v} = \begin{bmatrix}1\\1\\0\end{bmatrix}$

$\lambda=2 \quad \vec{w} = \begin{bmatrix}1\\1\\1\end{bmatrix}$

by the way, graphing calculators or software like Maple is good at these calculations:

Homework scratch work - Nov 14, 2001

```
> restart:with(linalg):

Warning, the protected names norm and trace have been redefined and unprotected

> A:=matrix(3,3,[7,-8,3,6,-7,3,2,-2,2]);
  #17, section 6.2
```

$$A := \begin{bmatrix} 7 & -8 & 3 \\ 6 & -7 & 3 \\ 2 & -2 & 2 \end{bmatrix}$$

```
> eigenvects(A);
```

[2, 1, {[1, 1, 1]}], [-1, 1, {[1, 1, 0]}], [1, 1, {[1, 0, -2]}]

eigenvalue

eigenbasis

$(\lambda-2)^{1}$ is factor in charact. poly. ("algebraic multiplicity")

So, for $P = \begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ we will get $P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = D$

check: rather then checking $P^{-1}AP = D$ I will check equiv eqtn $AP = PD$:

$$AP = \begin{bmatrix} 7 & -8 & 3 \\ 6 & -7 & 3 \\ 2 & -2 & 2 \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 2 \\ 0 & -1 & 2 \\ 2 & 0 & 2 \end{bmatrix}$$

$$PD = \begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 2 \\ 0 & -1 & 2 \\ 2 & 0 & 2 \end{bmatrix} \checkmark$$

8. $\det(A-\lambda I) = \det\begin{bmatrix} 1-\lambda & 1 & 0 & 1 \\ 0 & 1-\lambda & 1 & 1 \\ 0 & 0 & 2-\lambda & 1 \\ 0 & 0 & 0 & 2-\lambda \end{bmatrix} = (1-\lambda)^2(2-\lambda)^2$

$\lambda = 1$:
$$\left[\begin{array}{cccc|c} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \quad \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$v_4 = 0$
$v_3 = 0$
$v_2 = 0$
$v_1 = t$

$\vec{v} = t\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 1$ E-space $= \text{span}(\vec{e}_1)$

$\lambda = 2$:
$$\left[\begin{array}{cccc|c} -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$v_4 = 0$
$v_3 = t$
$v_2 = t$
$v_1 = t$

$\vec{v} = t\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = 2$ E-space $= \text{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}\right\}$

only 2 lin. ind. eigenvectors – need 4!

<u>A is NOT diagonalizable</u>

7.1 3. $t^2x'' + tx' + (t^2-1)x = 0 \quad \longleftrightarrow \quad \begin{array}{l} x_1 = x, \\ x_2 = x' \end{array}$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{1}{t}x_2 + \frac{(1-t^2)}{t^2}x_1 \end{bmatrix}$$

$x_2' = x'' = -\frac{1}{t}x' + \frac{(1-t^2)}{t^2}x$

or $\begin{bmatrix} x_1' \\ t^2 x_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ -tx_2 + (1-t^2)x_1 \end{bmatrix}$

8. $x'' + 3x' + 4x - 2y = 0$
$y'' + 2y' - 3x + y = \cos t$

let $x_1 = x$
$x_2 = x'$
$y_1 = y$
$y_2 = y'$

$$\begin{bmatrix} x_1' \\ x_2' \\ y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ -4x_1 - 3x_2 + 2y_1 \\ y_2 \\ 3x_1 - y_1 - 2y_2 + \cos t \end{bmatrix}$$

$x_2' = x'' = -3x' - 4x + 2y$
$y_2' = y'' = -2y' + 3x - y + \cos t$

18. $x' = -y$
$y' = 10x - 7y$

$\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \end{bmatrix}$

this one can be converted into a 2nd order eqtn:
$x' = -y$
so $x'' = -y' = -10x + 7y$
$= -10x - 7x'$

$x'' + 7x' + 10x = 0$

$x = e^{rt}$: $r^2 + 7r + 10 = 0$
$(r+5)(r+2) = 0$; $r = -2, -5$

$x = c_1e^{-5t} + c_2e^{-2t}$
$y = -x' = 5c_1e^{-5t} + 2c_2e^{-2t}$

$x(0) = 2 = c_1 + c_2$
$y(0) = -7 = 5c_1 + 2c_2$

$$c_1 = \frac{\begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix}} = \frac{11}{-3} = -\frac{11}{3}$$

$$c_2 = \frac{\begin{vmatrix} 1 & 2 \\ 5 & -7 \end{vmatrix}}{-3} = \frac{-17}{-3} = \frac{17}{3}$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = -\frac{11}{3}e^{-5t}\begin{bmatrix} 1 \\ 5 \end{bmatrix} + \frac{17}{3}e^{-2t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

<u>Note</u> $\left\{ \begin{array}{l} \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 10 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \\ \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \end{bmatrix} \end{array} \right.$

Could you do this using evals/evecs