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Math 2250-4
HW due 8/29

1.1 3, 4, 7 15, 16, 19, 20 27 28 33 35 36
1.2 5, 7 10 13 14 19 21 25 26 29 30 36
1.3 3, 6 8 11, 12, 16, 21, 24 32, 33.

1.1 7) $y''-2y'+2y=0$

$y_1 = e^x \cos x$

$y_1' = e^x\cos x - e^x \sin x = e^x(\cos x - \sin x)$ (product rule)

$y_1'' = e^x(\cos x - \sin x + (-\sin x - \cos x)) = -2e^x \sin x$

$y_1''-2y_1'+2y_1 = e^x\cos x\,(0-2+2) + e^x \sin x\,(-2+2+0) = 0.$ ✓

$2 \; (\; y_2 = e^x \sin x$

$-2 \; (\; y_2' = e^x(\sin x + \cos x)$

$+1 \; (\; y_2'' = e^x(\sin x + \cos x + \cos x - \sin x) = e^x 2\cos x$

$y_2''-2y_2'+2y_2 = e^x\cos x\,(2-2+0) + e^x\sin x\,(0-2+2) = 0$ ✓

16) $3y''+3y'-4y=0$

if $y=e^{rx}$

$y' = re^{rx}$

$y'' = r^2e^{rx}$

$3y''+3y'-4y = e^{rx}(3r^2+3r-4)=0$

must have $3r^2+3r-4=0$

$r = \frac{-3\pm\sqrt{9+48}}{6} = -\frac{1}{2} \pm \frac{\sqrt{57}}{6}$

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20) $y' = x-y$

$y(x) = Ce^{-x}+x-1$

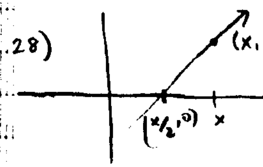
$y' = -Ce^{-x}+1$

$x-y = \cancel{x}-(Ce^{-x}+\cancel{x}-1) = -Ce^{-x}+1$)) equal, so $y'=x-y$

$y(0)=10 = Ce^0+0-1 = C-1$; $C=11$;

$\boxed{y(x) = 11e^{-x}+x-1}$

28)



$\frac{dy}{dx}$ = tang line slope

$= \frac{y-0}{x-x/2} = \frac{y}{x/2} = \frac{2y}{x}$

$\boxed{y' = \frac{2y}{x}}$

35) $\frac{dN}{dt}$ $\underbrace{\text{is prop. to}}_{=k\cdot}$ $\underbrace{\text{those who haven't heard rumor}}_{P-N}$

$\boxed{\frac{dN}{dt} = k(P-N)}$

36) $\frac{dN}{dt}$ $\underbrace{\text{is prop to}}_{=k\cdot}$ $\underbrace{\text{prod of N}}_{N}$ $\underbrace{\text{with \#not infected}}_{P-N}$

$\boxed{\frac{dN}{dt} = kN(P-N)}$

§1.2) 7) $\begin{cases} \frac{dy}{dx} = \frac{10}{x^2+1} \\ y(0)=0 \end{cases}$

$y = \int \frac{10}{x^2+1}\,dx = 10\arctan x + C$

$y(0) = 0 = 10\cdot 0 + C$; $C=0$

$\boxed{y = 10\tan^{-1}x}$

5.2 10) $\begin{cases} \frac{dy}{dx} = xe^{-x} \\ y(0)=1 \end{cases}$

$y = \int xe^{-x}\,dx + C$

$u = x \quad du = dx$
$dv = e^{-x}dx \quad v = -e^{-x}$

$\int u\,dv = uv - \int v\,du$

$y = -xe^{-x} - \int -e^{-x}dx$

$y = -xe^{-x} - e^{-x} + C$

$y(0) = 1 = 0 - 1 + C; \; C = 2$

$\boxed{y = 2 - e^{-x}(1+x)}$

14) $a(t) = 2t+1$

$v(t) = \int a(t)dt$

$v(t) = t^2 + t + C$

$v_0 = -7 = 0 + 0 + C$

so $v(t) = t^2 + t - 7$

$x(t) = \int v(t)dt = \frac{t^3}{3} + \frac{t^2}{2} - 7t + C$

$x_0 = 4 = 0 + 0 - 0 + C; \quad C = 4$

so $\boxed{x(t) = \frac{t^3}{3} + \frac{t^2}{2} - 7t + 4}$

25) $\frac{dv}{dt} = .12t^2 + .6t$

so $v = .04t^3 + .3t^2 + C$

$v_0 = 0 = C$

$v = .04t^3 + .3t^2$

so $x(t) = .01t^4 + .1t^3 + C$

$x_0 = 0 = C$

$\boxed{x(t) = .01t^4 + .1t^3}$

so $x(10) = (.01)(10^4) + (.1)10^3$

$\boxed{x(10) = 200 \text{ ft}}$

$v(10) = (.04)(10^3) + (.3)10^2$

$\boxed{v(10) = 70 \text{ ft/sec}}$

26) acceleration a is unknown.

$x''(t) = a$

$x'(t) = at + v_0$

$x(t) = \frac{1}{2}at^2 + v_0t + x_0$

told $v_0 = 88$ ft/sec

pick $x_0 = 0$

So $\boxed{x(t) = \frac{1}{2}at^2 + 88t}$

Let T be stopping time (skid end)

told: $x(T) = 176$

know $v(T) = x'(T) = 0$

$aT + 88 = 0;$

$T = -\frac{88}{a}$ (a is neg.)

$176 = x(T) = \frac{1}{2}a\left(-\frac{88}{a}\right)^2 + 88\left(-\frac{88}{a}\right)$

$\div 88$: $2 = \frac{1}{2a}\cdot 88 - \frac{88}{a} = -\frac{44}{a}$

26 cont'd. $\boxed{\begin{array}{l} \Rightarrow a = -22 \text{ ft/sec}^2 \\ \Rightarrow T = -\frac{88}{a} = 4 \text{ sec.} \end{array}}$

29) again, a is unknown but told that if $x_0 = 20$, $v_0 = 0$ then $x(2) = 0$

$x(t) = -\frac{1}{2}at^2 + v_0t + x_0$

$= -\frac{1}{2}at^2 + 20$

$x(2) = 0 = -\frac{1}{2}a\cdot 4 + 20 = -2a + 20; \quad a = 10$ ft/s²

so $\boxed{a = 10}$ if $x_0 = 200$, $v_0 = 0$

then $x(t) = -5t^2 + 200$

if $x(T) = 0$ then $-5T^2 + 200 = 0$

$T^2 = 40$

$\boxed{T = 2\sqrt{10} \text{ sec.}}$

$x'(t) = -10t$

$x'(T) = v(T) = -10(2\sqrt{10}) = -20\sqrt{10}$ ft/sec

$\boxed{\text{speed} = |x'(T)| = 20\sqrt{10} \text{ ft/s}}$

30) person's v_0 remains the same on Gzyx & earth.

on earth ball traj is

$x(t) = -\frac{1}{2}gt^2 + v_0t + x_0; \quad x_0 = 0$ (approximately)

$v(t) = -gt + v_0$ at max ht $v(T) = 0 = -gT + v_0$

$T = \frac{v_0}{g}$

told $x(T) = 144$

know T, can compute v_0:

$144 = x\left(\frac{v_0}{g}\right) = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\left(\frac{v_0}{g}\right)$

$144 = \frac{v_0^2}{2g} \Rightarrow v_0 = 12\sqrt{2g} \quad g \approx 32$ ft/sec²

$v_0 \approx 96$ ft/sec $(2g \approx 64)$

so on Gzyx, $x(t) = -\frac{1}{2}at^2 + 96t$

$x'(t) = -at + 96 = 0$ at $T = \frac{96}{a} = 9.6$

$x(T) = -\frac{1}{2}(10)(9.6)^2 + 96\times 9.6$

$= \frac{1}{2}96(9.6) \approx \boxed{460.8 \text{ ft}}$

check with energy arg.: PE + KE = const ⇒ same init. KE on earth & Gzyx; at max all energy is PE

$mgh_E = mah_G$

$\Rightarrow h_G = \frac{g}{a}h_E$

$= (3.2)h_E$

≈ 460.8 ✓